Examples 5: Complex Numbers 1

1. Express the following in the form $z = a + ib$ (where a and b are real numbers) and write down the complex conjugate, modulus and principal argument of each of the resulting complex numbers.

   (a) $(3 + 4i) + (2 + 2i)$

   (b) $(2 + 3i)(3 - 2i)$

   (c) $1/i$

   (d) $(1 + i)^2/(1 - i)$

2. Plot the following on the Argand diagram

   (a) $2 + 3i$, (b) $-2 - i$, (c) $4 - 3i$, (d) $-3 + 2i$.

3. Given that x and y are real, write

$$\frac{x}{2+i} - \frac{2y}{1-i} + 3i$$

   in the form $a + ib$ (where a and b are real). Hence find the values of x and y which satisfy the equation

$$\frac{x}{2+i} - \frac{2y}{1-i} + 3i = 0.$$

4. Find the locus of the complex numbers z that satisfy $|z + 2i - 3| = |z + 3i|$. Find the complex number that lies on this locus and has $\arg z = \pi/4$.

5. Find the roots of the equation

$$z^3 - (2 + i)z^2 + z - 2 - i = 0.$$

   [Hint: look for an 'obvious' root.]

Answers

1. (a) $z = 5 + 6i,\ \bar{z} = 5 - 6i,\ |z| = |\bar{z}| = \sqrt{61},\ \mathrm{Arg}z = 0.8761,\ \mathrm{Arg}\bar{z} = -0.8761$
   (b) $z = 12 + 5i,\ \bar{z} = 12 - 5i,\ |z| = |\bar{z}| = 13,\ \mathrm{Arg}z = 0.3948,\ \mathrm{Arg}\bar{z} = -0.3948$
   (c) $z = -i,\ \bar{z} = i,\ |z| = |\bar{z}| = 1,\ \mathrm{Arg}z = -\pi/2,\ \mathrm{Arg}\bar{z} = \pi/2$
   (d) $z = -1 + i,\ \bar{z} = -1 - i,\ |z| = |\bar{z}| = \sqrt{2},\ \mathrm{Arg}z = 3\pi/4,\ \mathrm{Arg}\bar{z} = -3\pi/4.$

3. $(2x/5 - y) + i(3 - y - x/5)$; $x = 5, y = 2$.

4. Locus: $y = -3x + 2$. $z = (1 + i)/2$

5. $2 + i,\ \pm i$