

## MATRICES: GAUSSIAN ELIMINATION

**5 minute review.** Remind students how to use Gaussian elimination to solve systems of equations and invert matrices. This is best done using the warm-up questions.

**Class warm-up.** Find the inverse of the matrix  $A$  below using Gaussian elimination.

$$A = \begin{bmatrix} 1 & 5 \\ -2 & 5 \end{bmatrix}$$

Solve the system of equations below using Gaussian elimination.

$$\begin{aligned}x_1 - x_2 + 2x_3 &= 0 \\3x_1 + x_3 &= -7 \\x_2 + 4x_3 &= 9\end{aligned}$$

**Problems.** Choose from the below.

1. **Gaussian elimination versus matrix methods: systems of equations.**

For each of the systems below, solve using both Gaussian elimination and matrix methods. Which do you prefer and why? Perhaps race your neighbour and compare results.

(a)

$$\begin{aligned}4x_2 + 2x_3 &= 1 \\x_1 - x_3 &= 0 \\-3x_1 + 8x_2 + 3x_3 &= 5\end{aligned}$$

(b)

$$\begin{aligned}4x_1 - x_2 + 5x_3 &= 10 \\3x_1 + 2x_2 + 6x_3 &= -1 \\-x_1 + 4x_2 &= 0;\end{aligned}$$

(c)

$$\begin{aligned}2x_1 + 2x_2 + 19x_3 &= 2 \\5x_1 + x_2 - x_3 &= 3 \\4x_1 + 7x_2 + 6x_3 &= 10\end{aligned}$$

2. **Gaussian elimination versus matrix methods: inverses.** For each matrix, find the inverse using both Gaussian elimination and matrix methods. Once again, which do you prefer and why?

$$A = \begin{bmatrix} 1 & 6 & 0 \\ 3 & 3 & -3 \\ 0 & 7 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 & -2 & 1 \\ 3 & 0 & 0 & 4 \\ 1 & 1 & 1 & -1 \\ 2 & 0 & -2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 3 & 1 & 5 \\ 1 & 0 & 3 & 1 \\ 0 & 2 & -3 & 2 \\ 0 & 2 & 3 & 1 \end{bmatrix}.$$

For the warm-up, start from the augmented matrix  $\left[ \begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ -2 & 5 & 0 & 1 \end{array} \right]$  and carry out row operations with the aim of making the first two columns into the 2x2 identity matrix to get  $A^{-1} = \frac{1}{15} \begin{bmatrix} 5 & -5 \\ 2 & 1 \end{bmatrix}$ .

Starting from the augmented matrix  $\left[ \begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 3 & 0 & 1 & -7 \\ 0 & 1 & 4 & 9 \end{array} \right]$ , carry out row operations to make the first three columns into an upper triangular matrix to find the solution  $[-3 \ 1 \ 2]^T$ .

### Selected answers and hints.

- The Gaussian elimination method is described above; for the other method we find the inverse.

$$(a) \ A^{-1} = \frac{1}{16} \begin{bmatrix} 8 & 4 & -4 \\ 0 & 6 & 2 \\ 8 & -12 & -4 \end{bmatrix}. \text{ The solution is } \left[ \frac{-3}{4} \ \frac{5}{8} \ \frac{-3}{4} \right]^T.$$

$$(b) \ A^{-1} = -\frac{1}{20} \begin{bmatrix} -24 & 20 & -16 \\ -6 & 5 & -9 \\ 14 & -15 & 11 \end{bmatrix}. \text{ The solution is } \left[ 13 \ \frac{13}{4} \ \frac{-31}{4} \right]^T.$$

$$(c) \ A^{-1} = \frac{1}{547} \begin{bmatrix} 13 & 121 & -21 \\ -34 & -64 & 97 \\ 31 & -6 & -8 \end{bmatrix}. \text{ The solution is } \left[ \frac{179}{547} \ \frac{710}{547} \ \frac{-36}{547} \right]^T.$$

- (a)

$$A^{-1} = \frac{1}{6} \begin{bmatrix} 24 & -6 & -18 \\ -3 & 1 & 3 \\ 21 & -7 & -15 \end{bmatrix}$$

- (b)

$$B^{-1} = \frac{1}{42} \begin{bmatrix} -8 & -10 & 48 & 32 \\ 13 & 11 & -36 & -31 \\ 1 & 17 & -6 & -25 \\ 6 & 18 & -36 & -24 \end{bmatrix}$$

- (c)

$$C^{-1} = \begin{bmatrix} 18 & -35 & -28 & 1 \\ 9 & -18 & -14 & 1 \\ -2 & 4 & 3 & 0 \\ -12 & 24 & 19 & -1 \end{bmatrix}$$

For more details, start a thread on the discussion board.