MATRICES: GAUSSIAN ELIMINATION

5 minute review. Remind students how to use Gaussian elimination to solve systems of equations and invert matrices. This is best done using the warm-up questions.

Class warm-up. Find the inverse of the matrix A below using Gaussian elimination.

$$A = \begin{bmatrix} 1 & 5 \\ -2 & 5 \end{bmatrix}$$

Solve the system of equations below using Gaussian elimination.

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 0 \\ 3x_1 + x_3 &= -7 \\ x_2 + 4x_3 &= 9 \end{aligned}$$

Problems. Choose from the below.

1. **Gaussian elimination versus matrix methods: systems of equations**. For each of the systems below, solve using both Gaussian elimination and matrix methods. Which do you prefer and why? Perhaps race your neighbour and compare results.

   (a)

   $$\begin{aligned} 4x_2 + 2x_3 &= 1 \\ x_1 - x_3 &= 0 \\ -3x_1 + 8x_2 + 3x_3 &= 5 \end{aligned}$$

   (b)

   $$\begin{aligned} 4x_1 - x_2 + 5x_3 &= 10 \\ 3x_1 + 2x_2 + 6x_3 &= -1 \\ -x_1 + 4x_2 &= 0; \end{aligned}$$

   (c)

   $$\begin{aligned} 2x_1 + 2x_2 + 19x_3 &= 2 \\ 5x_1 + x_2 - x_3 &= 3 \\ 4x_1 + 7x_2 + 6x_3 &= 10 \end{aligned}$$

2. **Gaussian elimination versus matrix methods: inverses**. For each matrix, find the inverse using both Gaussian elimination and matrix methods. Once again, which do you prefer and why?

$$A = \begin{bmatrix} 1 & 6 & 0 \\ 3 & 3 & -3 \\ 0 & 7 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 & -2 & 1 \\ 3 & 0 & 0 & 4 \\ 1 & 1 & 1 & -1 \\ 2 & 0 & -2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 3 & 1 & 5 \\ 1 & 0 & 3 & 1 \\ 0 & 2 & -3 & 2 \\ 0 & 2 & 3 & 1 \end{bmatrix}.$$

For the warm-up, start from the augmented matrix $\left[\begin{array}{cc|cc} 1 & 5 & 1 & 0 \\ -2 & 5 & 0 & 1 \end{array}\right]$ and carry out row operations with the aim of making the first two columns into the 2x2 identity matrix to get $A^{-1} = \frac{1}{15}\begin{bmatrix} 5 & -5 \\ 2 & 1 \end{bmatrix}$.

Starting from the augmented matrix $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 3 & 0 & 1 & -7 \\ 0 & 1 & 4 & 9 \end{array}\right]$, carry out row operations to make the first three columns into an upper triangular matrix to find the solution $[-3 \quad 1 \quad 2]^T$.

Selected answers and hints.

1. The Gaussian elimination method is described above;for the other method we find the inverse.
   (a) $A^{-1} = \frac{1}{16}\begin{bmatrix} 8 & 4 & -4 \\ 0 & 6 & 2 \\ 8 & -12 & -4 \end{bmatrix}$. The solution is $[\frac{-3}{4} \quad \frac{5}{8} \quad \frac{-3}{4}]^T$.
   (b) $A^{-1} = -\frac{1}{20}\begin{bmatrix} -24 & 20 & -16 \\ -6 & 5 & -9 \\ 14 & -15 & 11 \end{bmatrix}$. The solution is $[13 \quad \frac{13}{4} \quad \frac{-31}{4}]^T$.
   (c) $A^{-1} = \frac{1}{547}\begin{bmatrix} 13 & 121 & -21 \\ -34 & -64 & 97 \\ 31 & -6 & -8 \end{bmatrix}$. The solution is $[\frac{179}{547} \quad \frac{710}{547} \quad \frac{-36}{547}]^T$.
2. (a)
$$A^{-1} = \frac{1}{6}\begin{bmatrix} 24 & -6 & -18 \\ -3 & 1 & 3 \\ 21 & -7 & -15 \end{bmatrix}$$
   (b)
$$B^{-1} = \frac{1}{42}\begin{bmatrix} -8 & -10 & 48 & 32 \\ 13 & 11 & -36 & -31 \\ 1 & 17 & -6 & -25 \\ 6 & 18 & -36 & -24 \end{bmatrix}$$
   (c)
$$C^{-1} = \begin{bmatrix} 18 & -35 & -28 & 1 \\ 9 & -18 & -14 & 1 \\ -2 & 4 & 3 & 0 \\ -12 & 24 & 19 & -1 \end{bmatrix}$$

For more details, start a thread on the discussion board.