

## MATRICES: MORE DETERMINANTS, AND INVERSES

**5 minute review.** Remind students how to compute determinants for any  $n \times n$  matrix using any row/column. Discuss how one can use row and column operations to help simplify computing determinants. Discuss the definition of the inverse of a matrix and how to find the inverse of a  $2 \times 2$  matrix.

**Class warm-up.** Use row and column operations to find the determinant of the matrix below.

$$A = \begin{bmatrix} 1 & 5 & 6 & 0 \\ 3 & 2 & 1 & 10 \\ 7 & 2 & -1 & 4 \\ -5 & 0 & 1 & 3 \end{bmatrix}$$

Find the inverse of the matrix below.

$$B = \begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix}.$$

**Problems.** Choose from the below.

- Determinants using row/column operations.** Use row and column operations to help you compute the determinants of the following  $4 \times 4$  matrices:

$$A = \begin{bmatrix} 3 & 15 & -4 & 2 \\ 1 & 1 & 1 & 7 \\ 5 & -3 & -5 & 6 \\ -5 & 0 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 0 & 4 \\ 7 & 3 & -7 & 10 \\ -2 & 5 & 6 & -11 \\ -1 & 8 & 15 & 8 \end{bmatrix}, C = \begin{bmatrix} 3 & 2 & 1 & -3 \\ 5 & 3 & 0 & 9 \\ 4 & 8 & 2 & 5 \\ -7 & 11 & 3 & -7 \end{bmatrix}.$$

- A large determinant.** Use your method of choice to compute the determinant of

$$A = \begin{bmatrix} -12 & 1 & -8 & 16 & 5 & 6 \\ 5 & 8 & 3 & 9 & 1 & -27 \\ 21 & -1 & 2 & 15 & -3 & 24 \\ -7 & 2 & 10 & 1 & 9 & 11 \\ 2 & 3 & 7 & 8 & 4 & 6 \\ 21 & 3 & 6 & 1 & 2 & 10 \end{bmatrix}.$$

- Inverses of  $2 \times 2$  matrices.** Find the inverses of the following matrices:

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & -4 \\ -4 & 1 \end{bmatrix}, C = \begin{bmatrix} 4 & 3 \\ -8 & -6 \end{bmatrix}.$$

- Are inverses unique?** Can a matrix have two different inverses? Let  $A$  be a matrix and let  $B$  and  $C$  both be inverses. What does this mean and does  $B = C$ ? (Hint: use the definition, not the formula.)

For the warm-up,  $|A| = 426$  and  $B^{-1} = \frac{1}{16}B$ .

**Selected answers and hints.**

1.  $|A| = 3623, |B| = 1196, |C| = -265$

2.  $|A| = -840596 = -4 \cdot 31 \cdot 6779$

3.

$$A^{-1} = \frac{1}{23} \begin{bmatrix} 7 & -1 \\ 2 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-14} \begin{bmatrix} 1 & 4 \\ 4 & 2 \end{bmatrix}$$

$C^{-1}$  does not exist.

4. Inverses are unique; if  $B, C$  are both inverses for  $A$  then

$$AB = I, \quad BA = I, \quad AC = I, \quad CA = I,$$

so

$$B = BI = BAC = IC = C.$$

For more details, start a thread on the discussion board.