MATRICES: MORE DETERMINANTS, AND INVERSES

5 minute review. Remind students how to compute determinants for any $n \times n$ matrix using any row/column. Discuss how one can use row and column operations to help simplify computing determinants. Discuss the definition of the inverse of a matrix and how to find the inverse of a 2×2 matrix.

Class warm-up. Use row and column operations to find the determinant of the matrix below.

$$A = \begin{bmatrix} 1 & 5 & 6 & 0 \\ 3 & 2 & 1 & 10 \\ 7 & 2 & -1 & 4 \\ -5 & 0 & 1 & 3 \end{bmatrix}$$

Find the inverse of the matrix below.

$$B = \begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix}.$$

Problems. Choose from the below.

1. **Determinants using row/column operations.** Use row and column operations to help you compute the determinants of the following 4×4 matrices:
$$A = \begin{bmatrix} 3 & 15 & -4 & 2 \\ 1 & 1 & 1 & 7 \\ 5 & -3 & -5 & 6 \\ -5 & 0 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 0 & 4 \\ 7 & 3 & -7 & 10 \\ -2 & 5 & 6 & -11 \\ -1 & 8 & 15 & 8 \end{bmatrix}, C = \begin{bmatrix} 3 & 2 & 1 & -3 \\ 5 & 3 & 0 & 9 \\ 4 & 8 & 2 & 5 \\ -7 & 11 & 3 & -7 \end{bmatrix}.$$

2. **A large determinant.** Use your method of choice to compute the determinant of
$$A = \begin{bmatrix} -12 & 1 & -8 & 16 & 5 & 6 \\ 5 & 8 & 3 & 9 & 1 & -27 \\ 21 & -1 & 2 & 15 & -3 & 24 \\ -7 & 2 & 10 & 1 & 9 & 11 \\ 2 & 3 & 7 & 8 & 4 & 6 \\ 21 & 3 & 6 & 1 & 2 & 10 \end{bmatrix}.$$

3. **Inverses of 2×2 matrices.** Find the inverses of the following matrices:
$$A = \begin{bmatrix} 3 & 1 \\ -2 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & -4 \\ -4 & 1 \end{bmatrix}, C = \begin{bmatrix} 4 & 3 \\ -8 & -6 \end{bmatrix}.$$

4. **Are inverses unique?** Can a matrix have two different inverses? Let A be a matrix and let B and C both be inverses. What does this mean and does $B = C$? (Hint: use the definition, not the formula.)

For the warm-up, $|A| = 426$ and $B^{-1} = \frac{1}{16}B$.

Selected answers and hints.

1. $|A| = 3623, |B| = 1196, |C| = -265$
2. $|A| = -840596 = -4 \cdot 31 \cdot 6779$
3. $$A^{-1} = \frac{1}{23}\begin{bmatrix} 7 & -1 \\ 2 & 3 \end{bmatrix}$$
   $$B^{-1} = \frac{1}{-14}\begin{bmatrix} 1 & 4 \\ 4 & 2 \end{bmatrix}$$
   C^{-1} does not exist.
4. Inverses are unique; if B, C are both inverses for A then
   $$AB = I, \quad BA = I, \quad AC = I, \quad CA = I,$$
   so
   $$B = BI = BAC = IC = C.$$

For more details, start a thread on the discussion board.