SEMESTER 1 REVISION

Announcement. Introduce yourself as necessary, and tell students that there is a full-class lecture in Week 1 (which some students will already have attended). In it will be a (formative) test on the Semester 1 material, which won't count towards their module mark. All should attend.

5 minute review. Briefly talk students through what was covered in Semester 1 (without details). Those who didn't teach in Semester 1 can keep this very brief.

- Functions (curve sketching, binomial theorem, inverse functions, exponential & logarithms, trigonometric & hyperbolic functions);
- Differentiation (first principles, differentiation rules, parametric & implicit differentiation, partial differentiation);
- Series (Maclaurin & Taylor series, l'Hôpital's rule);
- Complex numbers (polar & exponential forms, Argand diagram, Euler's relation, de Moivre's theorem);
- Vectors (scalar product, vector product).

Class warm-ups. (a) Sketch the function $f(x) = (2x-3)/(2-x)+5$. What is the domain of $f(x)$? Find the inverse $f^{-1}(x)$ and give its domain and range.

(b) Solve $4e^{2x+1} - e^2 = 0$.

Problems. Choose from the below.

1. **Differentiation**.
   (a) Evaluate $\frac{d}{dx}\left(\frac{xe^x}{x+1}\right)$.
   (b) Find $\frac{dy}{dx}$ given $x = 2t/(1+t^2)$ and $y = t^2/(1+t^2)$.
   (c) Differentiate x^n from first principles, where $n > 1$ is an integer.
2. **Series**.
   (a) Find the first four non-zero terms of the Maclaurin series for $\sin(2x+2)$.
   (b) Use l'Hopital's rule to find, $\lim_{x\to 0} \frac{\cos(x)-1}{x^2}$.
3. **Complex numbers**.
   (a) Find the real and imaginary parts of $z = (3+i)/(1+3i)$.
   (b) Write down $z = 1 + i\sqrt{3}$ in its polar and exponential forms, and plot it on the Argand diagram.
   (c) Use de Moivre's theorem to find $\sin(3\theta)$ in terms of powers of $\sin(\theta)$, and $\cos(3\theta)$ in terms of powers of $\cos(\theta)$.
4. **Vectors**.
   (a) Find the constants α and β, given that the vector $\mathbf{r} = (\alpha, 4, \beta)$ is perpendicular to each of the vectors $\mathbf{a} = (1,2,3)$ and $\mathbf{b} = (-1,1,1)$.
   (b) If $\mathbf{a} = (2,1,0)$, $\mathbf{b} = (3,5,2)$ and $\mathbf{c} = (1,1,-1)$, verify that $(\mathbf{a}\times\mathbf{b})\times\mathbf{c} = (\mathbf{a}.\mathbf{c})\mathbf{b} - (\mathbf{b}.\mathbf{c})\mathbf{a}$.

For the warm-ups:

(a) $f(x)$ has domain $x \neq 2$, and $f^{-1}(x) = (2x-7)/(x-3)$ for $x \neq 3$, with range $y \in \mathbb{R}$, $y \neq 2$.

(b) $x = (1 - \ln(4))/2$.

Selected answers and hints.

1. (a) The derivative is $e^x - (xe^x)/(x+1)^2$.
   (b) $\frac{dy}{dx} = t/(1-t^2)$.
   (c) $\frac{d}{dx}(x^n) = nx^{n-1}$, as well known. The working relies on the binomial expansion of $(x+h)^n$.
2. (a) $\sin(2x+2) = \sin(2) + 2x\cos(2) - 2x^2\sin(2) - (4/3)x^3\cos(2) + ...$
   (b) The limit is $-1/2$.
3. (a) $\mathrm{Re}(z) = 3/5$, $\mathrm{Im}(z) = -4/5$.
   (b) $z = 2(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})) = 2e^{i\frac{\pi}{3}}$.
   (c) By using the fact that $(\cos\theta + i\sin(\theta))^3 = \cos(3\theta) + i\sin(3\theta)$ and comparing real and imaginary parts, one finds that $\sin(3\theta) = 3\sin(\theta) - 4\sin^3(\theta)$ and $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$.
4. (a) $\alpha = 1, \beta = -3$.
   (b) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = (-3, 9, 6)$.

For more details, start a thread on the discussion board.