

VECTORS

5 minute review. Recap the difference between scalars (numbers) and vectors (arrows). Remind them about magnitudes, and how addition and scalar multiplication works. Please also emphasise that vectors should be underlined in written work.

Class warm-up. For points P and Q , we write \overrightarrow{PQ} for the vector which starts at P and ends at Q . Let $ABCDEF$ be a regular hexagon in which \overrightarrow{BC} corresponds to a vector \mathbf{b} and \overrightarrow{FC} corresponds to $2\mathbf{a}$. Express \overrightarrow{AB} , \overrightarrow{CD} and \overrightarrow{BE} in terms of \mathbf{a} and \mathbf{b} . (You don't need to be too formal in justifying answers; pictures will suffice.)

Problems. Choose from the below.

1. **Directions.** If \mathbf{a} represents a velocity 2ms^{-1} east and \mathbf{b} represents a velocity $2\sqrt{2}\text{ms}^{-1}$ north-west, determine the velocities represented by

$$-\mathbf{a}; -2\mathbf{b}; \mathbf{a} + \mathbf{b}.$$

2. **Magnitudes.** The angle between the vectors \mathbf{a} and \mathbf{b} is $\frac{\pi}{3}$, and $|\mathbf{a}| = |\mathbf{b}| = 3$. Draw diagrams showing $\mathbf{a} - \mathbf{b}$ and $\mathbf{a} + \mathbf{b}$, and use them to find $|\mathbf{a} - \mathbf{b}|$ and $|\mathbf{a} + \mathbf{b}|$.
3. **Unit vectors.** With reference to the standard 3-dimensional coordinate system, let A be the point $(2, 1, 0)$, B be $(-1, -1, 2)$ and O be the origin, $(0, 0, 0)$. For each part, give the coordinates of a point P such that

(a) \overrightarrow{OP} is a unit vector in the direction of \overrightarrow{OA} ;

(b) \overrightarrow{OP} is a unit vector in the direction of \overrightarrow{OB} ;

(c) \overrightarrow{OP} is a unit vector in the direction of \overrightarrow{AB} .

4. **Parallelograms.** Points A, B, C, D (which may not lie in the same plane) are joined to form a quadrilateral. The mid-points of AB, BC, CD, DA are P, Q, R, S respectively. Use vectors to show that $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC}$. Find \overrightarrow{SR} , and show that $PQRS$ must be a parallelogram.

For the warm-up, $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{CD} = \mathbf{b} - \mathbf{a}$ and $\overrightarrow{BE} = 2(\mathbf{b} - \mathbf{a})$.

Selected answers and hints.

1. $-\mathbf{a}$ is 2ms^{-1} west; $-2\mathbf{b} = 4\sqrt{2}\text{ms}^{-1}$ south-east; $\mathbf{a} + \mathbf{b}$ is 2ms^{-1} north.
2. (a) $|\mathbf{a} - \mathbf{b}| = 3$ and $|\mathbf{a} + \mathbf{b}| = 3\sqrt{3}$.
3. (a) $P = (\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0)$;
(b) $P = (-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}})$;
(c) $P = (-\frac{3}{\sqrt{17}}, -\frac{2}{\sqrt{17}}, \frac{2}{\sqrt{17}})$.
4. $\overrightarrow{SR} = \frac{1}{2}\overrightarrow{AC} = \overrightarrow{PQ}$. Similarly, $\overrightarrow{PS} = \overrightarrow{QR}$.

For more details, start a thread on the discussion board.