

MACLAURIN AND TAYLOR SERIES

5 minute review. Recap the definitions of Maclaurin and Taylor series, drawing attention to the $x - a$ in the definition of the Taylor series at the point $x = a$.

- The Maclaurin series for $f(x)$ is $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$.
- The Taylor series for $f(x)$ at $x = a$ is $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$.

Class warm-up. Go through one or two of the exercises from Problem 1, with input from the class.

Problems. Choose from the below.

1. **Basic exercises.** Find the Maclaurin series of the following functions.

(a) $f(x) = \frac{1}{1+3x}$; (b) $g(x) = \sin x$; (c) $h(x) = \cos x$; (d) $j(x) = e^x$; (e) $k(x) = \cosh x$.

2. **Taylor series.** Is there a Maclaurin series for $\ln(x)$? Find the Taylor series for $\ln(x)$ at $x = 1$.

3. **Approximating mathematical constants.**

- (a) Put $x = 1$ in the Maclaurin series for e^x . How many terms do you need before you get something starting 2.718...?
- (b) Find the Maclaurin series for $\arctan x$. Which value of x will give $\pi/4$ as an output? Which value of x will give $\frac{\pi}{6}$ as an output? Using these values in the Maclaurin series, which finds π to 6 decimal places faster?
- (c) Find the Maclaurin series for $\arcsin x$. What value will be given when $x = \frac{1}{2}$? How fast does the series obtained find the digits of π ?

Selected answers and hints.

1. (a) $\frac{1}{1+3x} = (1+3x)^{-1} = 1 - 3x + 9x^2 - 27x^3 + 81x^4 + \dots$
 - (b) $\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots$;
 - (c) $\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots$;
 - (d) $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots$;
 - (e) $\cosh x = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \frac{1}{6!}x^6 + \dots$;
2. There is no Maclaurin series for $\ln x$ as it is undefined at $x = 0$. The Taylor series for $\ln(x)$ at $x = 1$ is
- $$\ln(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots$$
3. (b) $\arctan x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots$. Putting $x = 1$ will give $\frac{\pi}{4}$ as an output; putting $x = \frac{1}{\sqrt{3}}$ will give $\frac{\pi}{6}$.
 - (c) $\arcsin x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \dots$. Putting $x = \frac{1}{2}$ will give $\frac{\pi}{6}$.

For more details, start a thread on the discussion board.