

## INVERSE FUNCTION RULE AND CURVE SKETCHING

**5 minute review.** Briefly cover the inverse trigonometric functions and their derivatives (and remind students that that  $\sin^{-1} x$  does not mean  $\frac{1}{\sin x}$ ).

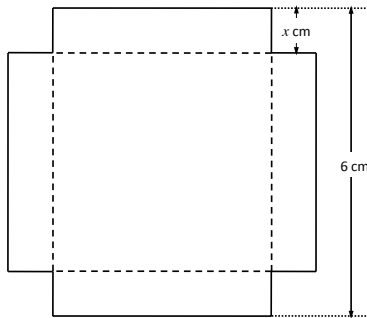
**Class warm-up.** Run through the proof that  $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$  with input from the class, asking for justification on the sign of the square-root in particular.

**Problems.** Choose from the below.

- More differentiation practice.** Differentiate the following, simplifying your answer as much as possible.

(a)  $f(x) = \cos(\sin x)$ ; (b)  $f(x) = \ln(3x \cos^2(2x))$ ; (c)  $f(x) = \ln \frac{\sin x + \cos x}{\sin x - \cos x}$ .

- Box revisited.** In week 1 we looked at an open box, made by folding a square piece of card as shown in the figure.



Prove using calculus that the maximum volume of this box occurs at  $x = 1$ .

- Differentiating**  $\operatorname{sech}^{-1} x$ .

(a) Differentiate  $y = \operatorname{sech} x (= \frac{1}{\cosh x})$ .

- (b) Sketch the graph of  $y = \cosh x$  and use this to sketch  $y = \operatorname{sech} x$ . Choose a domain on which  $\operatorname{sech} x$  is invertible. Hence sketch the graph of  $y = \operatorname{sech}^{-1} x$  and state its domain and range.

- (c) Using the inverse function rule, show that if  $y = \operatorname{sech}^{-1} x$  then

$$\frac{dy}{dx} = -\frac{1}{x\sqrt{1-x^2}}.$$

(Hint: use  $\cosh^2 x - \sinh^2 x = 1$  to find a relation linking  $\tanh^2 x$  and  $\operatorname{sech}^2 x$ .)

- Graph sketching.** Sketch the graph of  $y = f(x)$  for each of the following functions. You should think about domains, ranges, stationary points and crossings and ensure that you label all important points.

(a)  $f(x) = \frac{x}{(1+2x)^2}$ ; (b)  $f(x) = \left(\frac{x-3}{x^2+1}\right)^{\frac{1}{2}}$ ; (c)  $f(x) = \left(\frac{x-3}{x^2-1}\right)^{\frac{1}{2}}$ .

**Selected answers and hints.**

1. (a)  $f'(x) = -\cos x \cdot \sin(\sin x)$ ; (b)  $f'(x) = \frac{1}{x} - 4 \tan(2x)$ ; (c)  $f'(x) = 2 \sec(2x)$ .
2. Find a formula for the volume,  $V(x)$ , differentiate, find the stationary point(s) and check that  $x = 1$  gives a maximum.
3. (a)  $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$ .
4. (a)  $f(x) = \frac{x}{(1+2x)^2}$  is zero when  $x = 0$ , has an asymptote at  $x = -\frac{1}{2}$  and a maximum at  $(\frac{1}{2}, \frac{1}{8})$ ;  
 (b)  $f(x) = \left(\frac{x-3}{x^2+1}\right)^{\frac{1}{2}}$  has domain  $[3, \infty)$ , is zero when  $x = 3$  and has a maximum at  $x = 3 + \sqrt{10}$ ;  
 (c)  $f(x) = \left(\frac{x-3}{x^2-1}\right)^{\frac{1}{2}}$  has domain  $(-1, 1) \cup [3, \infty)$ , has vertical asymptotes at  $x = \pm 1$ , crosses the  $y$ -axis at  $y = \sqrt{3}$ , is zero when  $x = 3$  and has a maximum at  $x = 3 + 2\sqrt{2}$  and a minimum at  $x = 3 - 2\sqrt{2}$ .

For more details, start a thread on the discussion board.