DIFFERENTIATION RULES

Announcement. Remind students that there is a full-class lecture in Week 4 (MAS140 W1 SU AUD, MAS151 W1 DIA LT4, MAS152 M2 Church, MAS156(Elec) M5 DIA LT4, MAS156(Aero) Tu1 DIA LT1, MAS161 M5 DIA LT4). All should attend.

5 minute review. Remind the students of the addition, product, quotient and chain rules, the latter as both 'if $y = f(g(x))$ then $\frac{dy}{dx} = g'(x)f'(g(x))$' and 'if $y = f(u)$ with $u = u(x)$, then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$'.

Class warm-up. Find the derivatives of some of the functions from Problem 1. Give the students some time to work on them, then go through them together with input from the class.

Problems. Choose from the below.

1. **Differentiation practice**. Differentiate the following.

$$\text{(i) } y = \sin(x^2); \quad \text{(ii) } g(x) = e^{\cos x}.\sqrt{x};$$
$$\text{(iii) } h(x) = \tan(e^x) + e^{\tan x}; \quad \text{(iv) } j(x) = \frac{e^{-x^2}}{\sqrt{x+1}};$$
$$\text{(v) } k(x) = \sin(\sin(\sin(x))); \quad \text{(vi) } l(x) = e^x \sin(x)\sqrt{x}.$$

2. **Small changes**. Differentiation provides a formula for *small changes*: if h is small, then $f(x+h) \approx f(x) + hf'(x)$. Applied to the volume V as a function of radius r, for example, this gives $V(r + \delta r) \approx V(r) + \delta r \frac{dV}{dr}$.

   The volume of a sphere increases by 2%. Find (approximately) the corresponding percentage increase in surface area.

3. **The quotient rule again**.
   (a) Use the product rule to differentiate $y = e^x \times (\frac{1}{x})$.
   (b) Use the quotient rule to differentiate $y = \frac{e^x}{x}$.
   (c) Show that the derivative of $\frac{1}{f(x)}$ is $-\frac{f'(x)}{f(x)^2}$.
   (d) Let $g(x) = \frac{f_1(x)}{f_2(x)}$. Write this as the product $g(x) = f_1(x).\frac{1}{f_2(x)}$ and differentiate it with the product rule, using the result of part (c).

Selected answers and hints.

1. (i) $\frac{dy}{dx} = 2x\cos(x^2)$; (ii) $g'(x) = \frac{e^{\cos(x)}}{2\sqrt{x}}(1 - 2x\sin x)$;
   (iii) $h'(x) = e^x \sec^2(e^x) + e^{\tan x}\sec^2(x)$; (iv) $j'(x) = -\frac{(2x+1)^2}{2(x+1)^{\frac{3}{2}}}e^{-x^2}$;
   (v) $k'(x) = \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))$;
   (vi) $l'(x) = e^x\sin(x)\sqrt{x} + e^x\cos(x)\sqrt{x} + \frac{e^x\sin(x)}{2\sqrt{x}}$.

2. If δV is the change in the volume V, then $\delta V = V(r+\delta r) - V(r) \approx \delta r\frac{dV}{dr}$, using the small changes formula. Thus
   $$\frac{\delta V}{V} \approx \frac{\delta r\frac{dV}{dr}}{V} = \frac{\delta r.4\pi r^2}{\frac{4}{3}\pi r^3} = 3\frac{\delta r}{r}.$$
   Since the change in V is 2%, this gives $\frac{\delta r}{r} \approx \frac{0.02}{3}$.

   The same method applied for the surface area A gives $\frac{\delta A}{A} \approx 2\frac{\delta r}{r} \approx \frac{2\times 0.02}{3} \approx 0.013$ (2sf). This the increase in surface area is approximately 1.3%.

3. $\frac{d}{dx}(\frac{e^x}{x}) = \frac{(x-1)}{x^2}e^x$.

For more details, start a thread on the discussion board.