INVERSE FUNCTIONS, EXPONENTIALS AND LOGARITHMS

5 minute review. A brief reminder of inverse functions, exp and ln covering things like

- $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$;
- $y = f(x)$ and $y = f^{-1}(x)$ have graphs which are reflections in $y = x$;
- $\text{range}(f) = \text{domain}(f^{-1})$ and vice versa;
- e^x and $\ln x$ as an example of an inverse pair;
- $\ln(xy) = \ln x + \ln y$, $\ln(x/y) = \ln x - \ln y$, $\ln(x^p) = p \ln x$.

Class warm-up. Consider $f(x) = Ae^{x^2+1}$, where $A > 0$ is a constant. Is $f(x)$ odd, even or neither? Sketch $y = f(x)$. Identify a suitable domain on which $f(x)$ is invertible, and find its inverse function, f^{-1}. Sketch $y = f^{-1}(x)$ and give its domain and range.

Problems. Choose from the below.

1. **More inverse trigonometric functions**. Sketch the graph of $y = \sec(x) = \frac{1}{\cos(x)}$ using the techniques from Week 1. Identify a suitable domain on which $\sec(x)$ is invertible, and hence sketch a graph of $y = \text{arcsec}(x)$ stating its domain and range.

   Repeat with $y = \cot(x) = \frac{1}{\tan(x)}$.

2. **Some detective work**. Newton's law of cooling states that

   $$T = T_a + (T_0 - T_a)e^{-kt},$$

   where T is the temperature at time t of an object, T_a is the ambient (room) temperature, T_0 is temperature at $t = 0$, and k is a constant.

   A dead environmentalist is found in a room with a thermostat maintaining the ambient temperature at $18°C$. At midnight ($t = 0$) the temperature of the body is $26°C$. By 2am it has cooled to $22°C$. Find the value of k, sketch the graph of T as a function of time, and find the time of death, assuming a body temperature of $37°C$.

3. **Using logarithms**. By using logarithms and exponentials as needed, solve the following for x. (Remember that $\ln(xy) = \ln x + \ln y$, $\ln(x/y) = \ln x - \ln y$ and $\ln(x^p) = p \ln x$.)

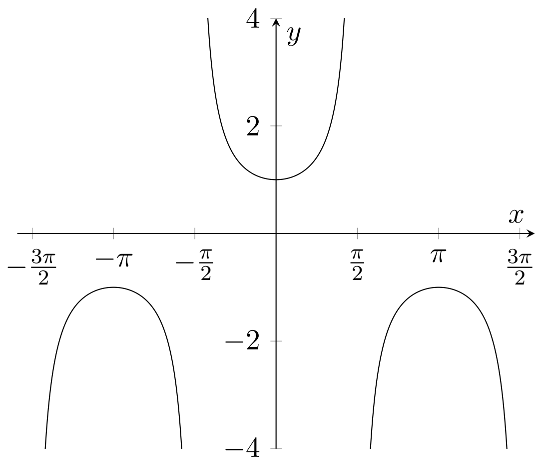
   (a) $e^{3x-2} = 23$; (b) $e^{2x^2+1} = 7$; (c) $e^{3\ln x} - e^6 = 0$; (d) $5e^{x+2} + e^2 = 34$;

   (e) $\ln(2x^6 - 2) = 10$; (f) $\dfrac{2^{\frac{1}{2}x+3}}{3^{\frac{2}{3}x+2}} = 3^{\frac{1}{3}x+1}$.

For the warm-up, $f(x)$ is even and invertible on the domain $[0, \infty)$. Inverting, $f^{-1}(x) = \sqrt{\ln x - \ln A - 1}$ with domain $[Ae, \infty)$ and range $[0, \infty)$.

Selected answers and hints.

1. The graph of $y = \sec x$ is below. A good choice of domain on which it is invertible is $[0, \frac{\pi}{2})$, or even $[0, \pi] \setminus \{\frac{\pi}{2}\}$ (i.e. $0 \leq x \leq \pi$, $x \neq \frac{\pi}{2}$ due to the discontinuity).



2. $\sim$ 9.30pm.

3. (a) $x = \frac{1}{3}(2 + \ln 23)$; (b) $x = \pm\frac{\sqrt{2}}{2}\sqrt{\ln 7 - 1}$; (c) $x = e^2$; (d) $x = \ln(34 - e^2) - \ln 5 - 2$; (e) $x = \pm\left(\frac{1}{2}e^{10} + 1\right)^{\frac{1}{6}}$; (f) $x = 6^{\frac{(\ln 3 - \ln 2)}{\ln 2 - 2\ln 3}}$.

For more details, start a thread on the discussion board.