CURVE SKETCHING

Welcome. Introduce yourself and remind them to attend the Week 1 lecture (if it hasn't already happened) to learn about how the course runs (MAS140 W1 DIA LT3, MAS151 M10 DIA LT1, MAS152 M1 SB LT1, MAS156(Aero) Tu1 DIA-LT4, MAS156(Elec) & MAS161 M5 DIA-LT4). Feel free to tell them the rough format of the course if comfortable, but don't get drawn into questions about particulars, as they will get these in the lecture.

5 minute review. Using the graph of $y = \frac{1}{x-1}$ (or any other suitably easy curve) as an example, remind students briefly

- what a graph is;
- the difference between plotting (by calculating) and sketching (by reasoning);
- the importance of labelling axes, crossing points and the curve itself;
- how to spot asymptotes, and how to reason about what happens near them (including at infinity).

Class warm-up. Asking for input at all suitable places, work through sketching the graph of $y = x\cos x$ on the board. Start by sketching the envelope $y = \pm x$, before discussing how the graph fluctuates within that envelope.

Problems. Choose from the below.

1. **General sketching**. Sketch
   (a) $y = \frac{x^2-x-2}{x}$;
   (b) $y = \frac{x}{x^2-x-2}$;
   (c) $y = \frac{x}{x^2-x-2} + 1$;
   (d) $y = \sec x \,(= \frac{1}{\cos x})$;
   (e) $y = \cot x \,(= \frac{1}{\tan x})$.
2. **Envelopes**. Sketch functions of the form $y = f(x)\cos(x)$ where
   (a) $f(x) = |x|$;
   (b) $f(x) = x^2$;
   (c) $f(x) = \cos(2x)$.
3. **Sketching a quotient**. Sketch the function $y = \frac{\sin x}{x}$. (Hints: it may help to start with $y = \pm\frac{1}{x}$ on the same axes; also, if you find the behaviour at $x = 0$ confusing, it may help to inspect values of x very close to zero).
4. **Lines and loops**.
   (a) Sketch $y = mx + c$, where (a) $m > 0$ and $c > 0$; (b) $m > 0$ and $c < 0$; (c) $m < 0$ and $c < 0$; (d) $m = 0$ and $c > 0$.
   (b) Sketch $x^2 + y^2 = 1$. (Hint: think Pythagoras.)
   (c) Sketch $x^4 + y^4 = 1$.
   (d) What happens to $x^{2n} + y^{2n} = 1$ as n gets larger?

Selected answers and hints.

1. Use Wolfram Alpha (`http://wolframalpha.com`) or similar to generate the curves.
2. As above.
3. As above.
4. (a) All of these are straight lines with gradient m and y-intercept c.

   (b) $x^2 + y^2 = 1$ is a circle of radius 1, centred on the origin.

   (d) $x^{2n} + y^{2n} = 1$ gets closer and closer to being a square as n gets larger.

For more details, start a thread on the discussion board.