CURVE SKETCHING

Welcome. For all except MAS152, this the first class for this module, so welcome the students, introduce yourself and remind them to attend the Week 1 lecture to learn about how the course runs (MAS140 W1 SU AUD, MAS151 W1 DIA LT4, MAS152 M2 Church, MAS156(Elec) M5 DIA LT4, MAS156(Aero) Tu1 DIA LT1, MAS161 M5 DIA LT4). Feel free to tell them the rough format of the course if comfortable, but don't get drawn into questions about particulars, as they will get these in the lecture (nb extra information provided separately for MAS156).

5 minute review. Using the graph of $y = \frac{1}{x-1}$ (or any other suitably easy curve) as an example, remind students briefly

- what a graph is;
- the difference between plotting (by calculating) and sketching (by reasoning);
- the importance of labelling axes, crossing points and the curve itself;
- how to spot asymptotes, and how to reason about what happens near them (including at infinity).

Class warm-up. Asking for input at all suitable places, work through sketching the graph of $y = x\cos x$ on the board. Start by sketching the envelope $y = \pm x$, before discussing how the graph fluctuates within that envelope.

Problems. Choose from the below.

1. **General sketching**. Sketch
   (a) $y = \frac{x^2-x-2}{x}$;
   (b) $y = \frac{x}{x^2-x-2}$;
   (c) $y = \frac{x}{x^2-x-2} + 1$;
   (d) $y = \sec x \,(= \frac{1}{\cos x})$;
   (e) $y = \cot x \,(= \frac{1}{\tan x})$.
2. **Envelopes**. Sketch functions of the form $y = f(x)\cos(x)$ where
   (a) $f(x) = |x|$;
   (b) $f(x) = x^2$;
   (c) $f(x) = \cos(2x)$.
3. **Sketching a quotient**. Sketch the function $y = \frac{\sin x}{x}$. (Hints: it may help to start with $y = \pm\frac{1}{x}$ on the same axes; also, if you find the behaviour at $x = 0$ confusing, it may help to inspect values of x very close to zero).
4. **Lines and loops**.
   (a) Sketch $y = mx + c$, where (a) $m > 0$ and $c > 0$; (b) $m > 0$ and $c < 0$; (c) $m < 0$ and $c < 0$; (d) $m = 0$ and $c > 0$.
   (b) Sketch $x^2 + y^2 = 1$. (Hint: think Pythagoras.)
   (c) Sketch $x^4 + y^4 = 1$.
   (d) What happens to $x^{2n} + y^{2n} = 1$ as n gets larger?

Selected answers and hints.

1. Use Wolfram Alpha (`http://wolframalpha.com`) or similar to generate the curves.
2. As above.
3. As above.
4. (a) All of these are straight lines with gradient m and y-intercept c.

   (b) $x^2 + y^2 = 1$ is a circle of radius 1, centred on the origin.

   (d) $x^{2n} + y^{2n} = 1$ gets closer and closer to being a square as n gets larger.

For more details, start a thread on the discussion board.