

# MAS152: Essential Mathematical Skills & Techniques

Prof Koji Ohkitani

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Monday 5th February 2018, 5pm  
Diamond LT1

**This semester**

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Some tutors may have changed. We hope this isn't too much of a disappointment!

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That lecture will cover exam technique.

# Discussion board

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Please remember the discussion board (found via the course webpage) which is by far the best place to ask questions.

# Class test

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Calculators are allowed, but not strictly necessary. Notes from last semester and smartphones should be put away!

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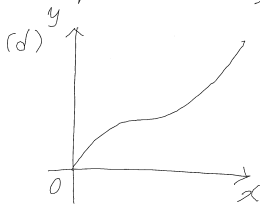
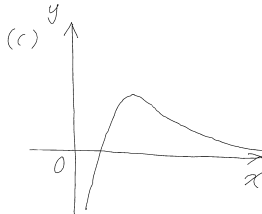
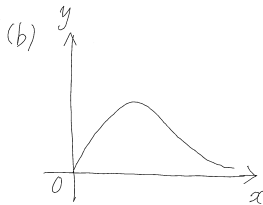
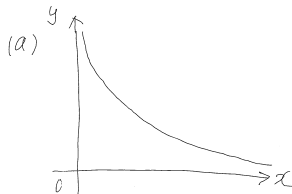
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Please do rough workings on another sheet of paper. At the end we will swap answer sheets and go through the questions.

# Question 1

Which sketch represents the function  $y = f(x) = \frac{\ln x}{x}$  best ?



## Question 2

The first four terms in the expansion of  $(1 - 2x)^{\frac{1}{2}}$  are

(a)  $1 - x + \frac{x^2}{2} - \frac{x^3}{2} + \dots$       (b)  $1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} + \dots$

(c)  $1 - x - \frac{x^2}{2} - \frac{x^3}{2} + \dots$       (d)  $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16} + \dots$



## Question 3

The complex number  $\frac{7 + 9i}{1 - 3i}$  is equal to

(a)  $\frac{17 - 6i}{5}$ ;      (b)  $\frac{-10 - 6i}{5}$ ;

(c)  $\frac{17 + 15i}{5}$ ;      (d)  $-2 + 3i$ .

## Question 4

The complex number  $z = -2 - i$  expressed in exponential form  $re^{i\theta}$ , with  $\theta$  expressed in radians, is equal to

(a)  $\sqrt{5}e^{-2.68i}$ ;      (b)  $5e^{-2.68i}$ ;

(c)  $\sqrt{5}e^{0.46i}$ ;      (d)  $5e^{0.46i}$ .

## Question 5

Which one of the following identities are *not* correct ?

(a)  $e^{ix} = \cos x + i \sin x$ ;      (b)  $e^x = \cosh x + \sinh x$ ;

(c)  $\cosh(ix) = \cos(x)$ ;      (d)  $\sinh(ix) = \sin(x)$ .

## Question 6

If  $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$  then  $\frac{dy}{dx}$  is

(a)  $\frac{1 + 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$ ;

(b)  $\frac{1 - 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$ ;

(c)  $\frac{-1 + 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$ ;

(d)  $\frac{-1 - 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$ .

## Question 7

If  $x$  and  $y$  are given in terms of a parameter  $t$  by  $x = e^{2t} + t$  and  $y = t - \ln t$ , then  $\frac{dy}{dx}$  is

(a)  $\frac{t-1}{t(e^{2t}+1)}$ ;      (b)  $\frac{t-1}{t(2e^{2t}+1)}$ ;

(c)  $\frac{2(t-1)}{t(e^{2t}+t^2)}$ ;      (d)  $\frac{t(e^{2t}+1)}{t-1}$ .

## Question 8

Given a function of two variables,  $f(x, y) = x^3y - 3y^3 + 2x^2$ , the first-order partial derivatives are

$$(a) \quad \frac{\partial f}{\partial x} = 4x, \quad \frac{\partial f}{\partial y} = -9y^2;$$

$$(b) \quad \frac{\partial f}{\partial x} = 3x^2y + 2x^2, \quad \frac{\partial f}{\partial y} = x^3 - 3y^3;$$

$$(c) \quad \frac{\partial f}{\partial x} = 3x^2y + 4x, \quad \frac{\partial f}{\partial y} = x^3 - 9y^2;$$

$$(d) \quad \frac{\partial f}{\partial x} = 3x^2y - 9y^2, \quad \frac{\partial f}{\partial y} = x^3 + 4x.$$

## Question 9

The limit  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$  is

(a)  $-1$ ;      (b)  $1$ ;

(c)  $\infty$ ;      (d)  $0$ .

## Question 10

When  $x = 2$ , the function  $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$  has

- (a) a local maximum;      (b) a local minimum;
- (c) a point of inflexion;      (d) a non-zero gradient.



## Question 11

The value of  $\mu$  for which the vector  $\mathbf{a} = (3, -1, 2)$  is perpendicular to the vector  $\mathbf{b} = (1, \mu, -1)$  is

(a)  $-1$ ;      (b)  $1$ ;

(c)  $5$ ;      (d)  $-5$ .

## Question 12

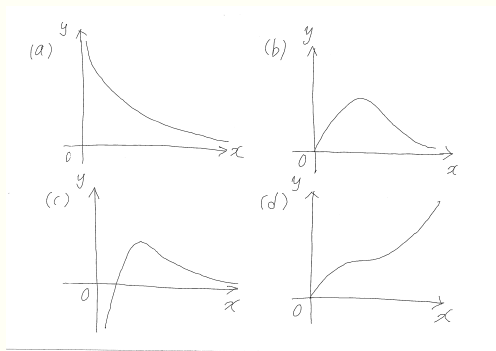
If  $\mathbf{a} = (1, -3, 2)$  and  $\mathbf{b} = (2, 1, -1)$ , then  $\mathbf{a} \times \mathbf{b}$  is

(a)  $(-1, -5, -7)$ ;      (b)  $(1, -5, 7)$ ;

(c)  $(1, 5, 7)$ ;      (d)  $-3$ .

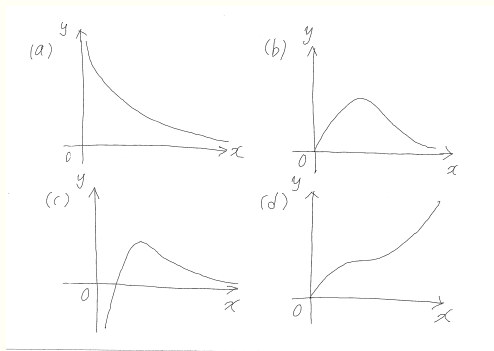
# Answers

# Question 1



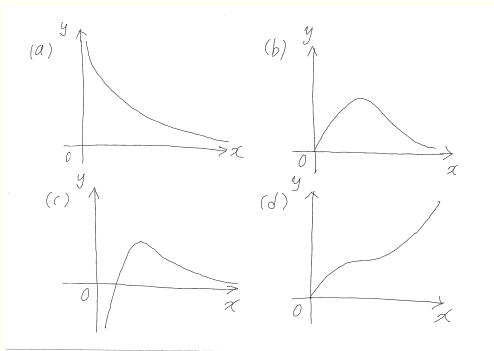
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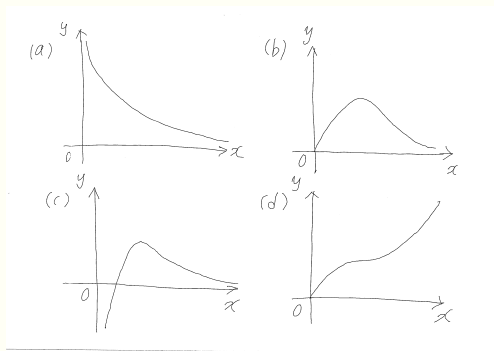
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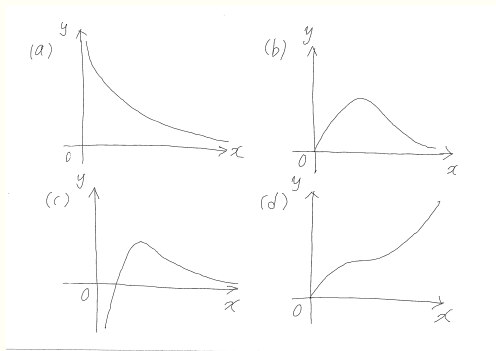
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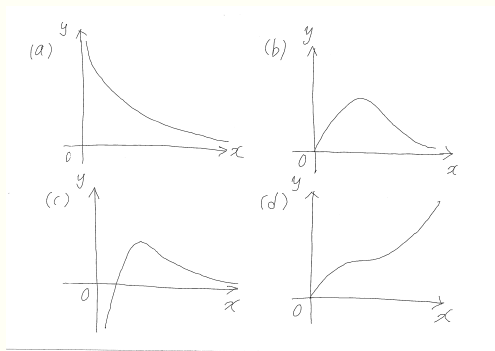


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Answer: **(c)**

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Answer: **(a)**

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Answer:



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Using the chain rule  $\frac{dy}{dx} = \left(\frac{dy}{du}\right) \cdot \left(\frac{du}{dx}\right)$ ,

$$\begin{aligned}\frac{dy}{dx} &= \left(\frac{1}{\frac{1}{x} + \cos(2x)}\right) \left(-\frac{1}{x^2} - 2\sin(2x)\right) \\ &= \left(\frac{x}{1 + x\cos(2x)}\right) \left(\frac{-1 - 2x^2\sin(2x)}{x^2}\right) \\ &= \frac{-1 - 2x^2\sin(2x)}{x + x^2\cos(2x)}.\end{aligned}$$

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Answer: **(d)**



## Question 7

If  $x$  and  $y$  are given in terms of a parameter  $t$  by  $x = e^{2t} + t$   
and  $y = t - \ln t$ , find  $\frac{dy}{dx}$ .

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Answer: **(b)**

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Answer: **(d)**

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Answer: **(a)**

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If  $\mathbf{a} = (1, -3, 2)$  and  $\mathbf{b} = (2, 1, -1)$ , find  $\mathbf{a} \times \mathbf{b}$ .

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$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

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Answer: **(c)**

**How did you do?**



Please return answer sheets to the person who wrote them.

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How good was your score?

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**That's it!**

See you in week 8.