MAS152: Essential Mathematical Skills & Techniques

Prof Koji Ohkitani
mas-engineering@sheffield.ac.uk

Monday 5th February 2018, 5pm
Diamond LT1

This semester

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Some tutors may have changed. We hope this isn't too much of a disappointment!

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- Monday 16th April (week 8), 5pm, Diamond LT4

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That lecture will cover exam technique.

Discussion board

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Please remember the discussion board (found via the course webpage) which is by far the best place to ask questions.

Class test

What follows is a test on Semester 1 material.

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Calculators are allowed, but not strictly necessary. Notes from last semester and smartphones should be put away!

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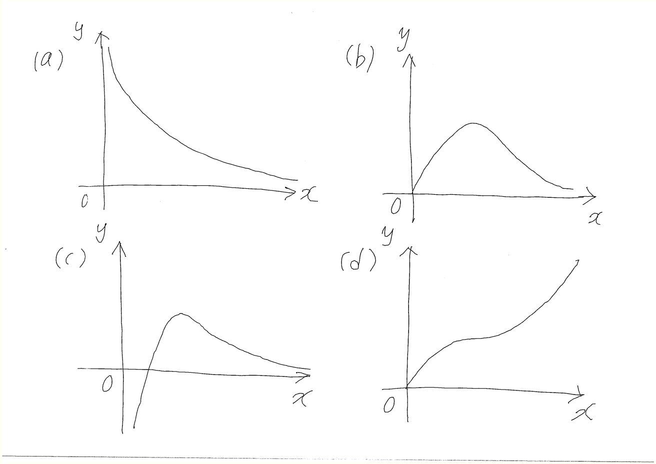
Please do rough workings on another sheet of paper. At the end we will swap answer sheets and go through the questions.

Question 1

Which sketch represents the function $y = f(x) = \frac{\ln x}{x}$ best ?



Question 2

The first four terms in the expansion of $(1-2x)^{\frac{1}{2}}$ are

(a) $1 - x + \frac{x^2}{2} - \frac{x^3}{2} + \dots$

(b) $1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} + \dots$

(c) $1 - x - \frac{x^2}{2} - \frac{x^3}{2} + \dots$

(d) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16} + \dots$

Question 3

The complex number $\dfrac{7+9i}{1-3i}$ is equal to

(a) $\dfrac{17-6i}{5}$; (b) $\dfrac{-10-6i}{5}$;

(c) $\dfrac{17+15i}{5}$; (d) $-2+3i$.

Question 4

The complex number $z = -2 - i$ expressed in exponential form $re^{i\theta}$, with θ expressed in radians, is equal to

(a) $\sqrt{5}e^{-2.68i}$; (b) $5e^{-2.68i}$;

(c) $\sqrt{5}e^{0.46i}$; (d) $5e^{0.46i}$.

Question 5

Which one of the following identities are *not* correct ?

(a) $e^{ix} = \cos x + i \sin x$; (b) $e^{x} = \cosh x + \sinh x$;

(c) $\cosh(ix) = \cos(x)$; (d) $\sinh(ix) = \sin(x)$.

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then $\frac{dy}{dx}$ is

(a) $\dfrac{1 + 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$;

(b) $\dfrac{1 - 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$;

(c) $\dfrac{-1 + 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$;

(d) $\dfrac{-1 - 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$.

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{t-1}{t\left(e^{2t}+1\right)}$; (b) $\dfrac{t-1}{t\left(2e^{2t}+1\right)}$;

(c) $\dfrac{2\left(t-1\right)}{t\left(e^{2t}+t^2\right)}$; (d) $\dfrac{t\left(e^{2t}+1\right)}{t-1}$.

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, the first-order partial derivatives are

(a) $\dfrac{\partial f}{\partial x} = 4x, \quad \dfrac{\partial f}{\partial y} = -9y^2$;

(b) $\dfrac{\partial f}{\partial x} = 3x^2y + 2x^2, \quad \dfrac{\partial f}{\partial y} = x^3 - 3y^3$;

(c) $\dfrac{\partial f}{\partial x} = 3x^2y + 4x, \quad \dfrac{\partial f}{\partial y} = x^3 - 9y^2$;

(d) $\dfrac{\partial f}{\partial x} = 3x^2y - 9y^2, \quad \dfrac{\partial f}{\partial y} = x^3 + 4x$.

Question 9

The limit $\lim_{x \to 0} \dfrac{\sin^2 x}{x^2 - x}$ is

(a) -1; (b) 1;

(c) ∞; (d) 0.

Question 10

When $x = 2$, the function $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ has

(a) a local maximum;

(b) a local minimum;

(c) a point of inflexion;

(d) a non-zero gradient.

Question 11

The value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$ is

(a) -1; (b) 1;

(c) 5; (d) -5.

Question 12

If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, then $\mathbf{a} \times \mathbf{b}$ is

(a) $(-1, -5, -7)$; (b) $(1, -5, 7)$;

(c) $(1, 5, 7)$; (d) -3.

Answers

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Answer: **(c)**

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Answer: **(c)**

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Answer: **(a)**

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Answer: **(d)**

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Answer: **(d)**

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Hence $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

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Answer: **(b)**

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Answer: **(c)**

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Answer: **(d)**

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Answer: **(a)**

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Answer: **(b)**

Question 12

If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, find $\mathbf{a} \times \mathbf{b}$.

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$$\mathbf{a} \times \mathbf{b} \quad = \quad \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

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\end{aligned}$$

Answer: **(c)**

How did you do?

Please return answer sheets to the person who wrote them.

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How good was your score?

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1. 0–2: A lot of work needed!

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5. 11 or 12 : EXCELLENT!!

That's it!

See you in week 8.