

MAS152: Essential Mathematical Skills & Techniques

Prof Koji Ohkitani

mas-engineering@sheffield.ac.uk

Monday 5th February 2018, 5pm
Diamond LT1

This semester

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Some tutors may have changed. We hope this isn't too much of a disappointment!

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That lecture will cover exam technique.

Discussion board

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Please remember the discussion board (found via the course webpage) which is by far the best place to ask questions.

Class test

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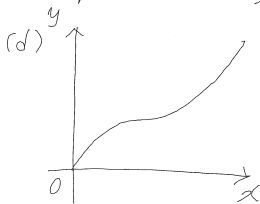
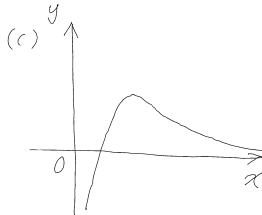
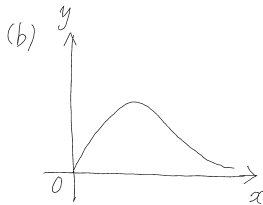
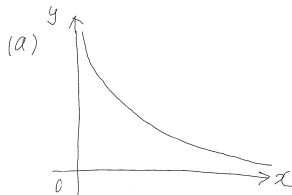
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Please do rough workings on another sheet of paper. At the end we will swap answer sheets and go through the questions.

Question 1

Which sketch represents the function $y = f(x) = \frac{\ln x}{x}$ best ?



Question 2

The first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$ are

(a) $1 - x + \frac{x^2}{2} - \frac{x^3}{2} + \dots$ (b) $1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} + \dots$

(c) $1 - x - \frac{x^2}{2} - \frac{x^3}{2} + \dots$ (d) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16} + \dots$

Question 3

The complex number $\frac{7 + 9i}{1 - 3i}$ is equal to

(a) $\frac{17 - 6i}{5}$; (b) $\frac{-10 - 6i}{5}$;

(c) $\frac{17 + 15i}{5}$; (d) $-2 + 3i$.

Question 4

The complex number $z = -2 - i$ expressed in exponential form $re^{i\theta}$, with θ expressed in radians, is equal to

(a) $\sqrt{5}e^{-2.68i}$; (b) $5e^{-2.68i}$;

(c) $\sqrt{5}e^{0.46i}$; (d) $5e^{0.46i}$.

Question 5

Which one of the following identities are *not* correct ?

(a) $e^{ix} = \cos x + i \sin x$; (b) $e^x = \cosh x + \sinh x$;

(c) $\cosh(ix) = \cos(x)$; (d) $\sinh(ix) = \sin(x)$.

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then $\frac{dy}{dx}$ is

(a) $\frac{1 + 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$;

(b) $\frac{1 - 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$;

(c) $\frac{-1 + 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$;

(d) $\frac{-1 - 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$.

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, then $\frac{dy}{dx}$ is

(a) $\frac{t-1}{t(e^{2t}+1)}$; (b) $\frac{t-1}{t(2e^{2t}+1)}$;

(c) $\frac{2(t-1)}{t(e^{2t}+t^2)}$; (d) $\frac{t(e^{2t}+1)}{t-1}$.

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, the first-order partial derivatives are

$$(a) \quad \frac{\partial f}{\partial x} = 4x, \quad \frac{\partial f}{\partial y} = -9y^2;$$

$$(b) \quad \frac{\partial f}{\partial x} = 3x^2y + 2x^2, \quad \frac{\partial f}{\partial y} = x^3 - 3y^3;$$

$$(c) \quad \frac{\partial f}{\partial x} = 3x^2y + 4x, \quad \frac{\partial f}{\partial y} = x^3 - 9y^2;$$

$$(d) \quad \frac{\partial f}{\partial x} = 3x^2y - 9y^2, \quad \frac{\partial f}{\partial y} = x^3 + 4x.$$

Question 9

The limit $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$ is

(a) -1 ; (b) 1 ;

(c) ∞ ; (d) 0 .

Question 10

When $x = 2$, the function $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ has

- (a) a local maximum; (b) a local minimum;
- (c) a point of inflexion; (d) a non-zero gradient.

Question 11

The value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$ is

(a) -1 ; (b) 1 ;

(c) 5 ; (d) -5 .

Question 12

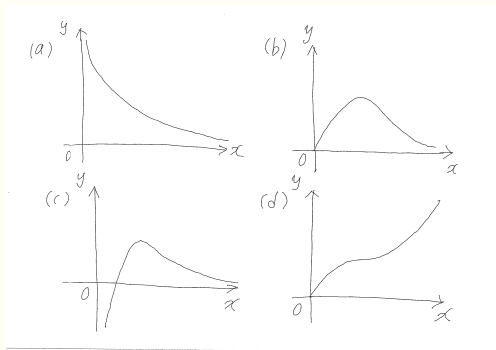
If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, then $\mathbf{a} \times \mathbf{b}$ is

(a) $(-1, -5, -7)$; (b) $(1, -5, 7)$;

(c) $(1, 5, 7)$; (d) -3 .

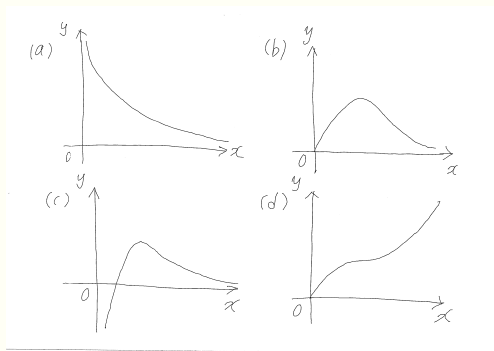
Answers

Question 1



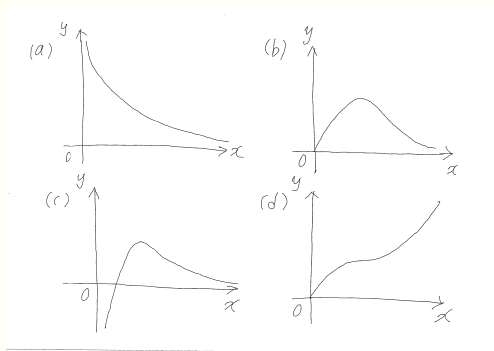
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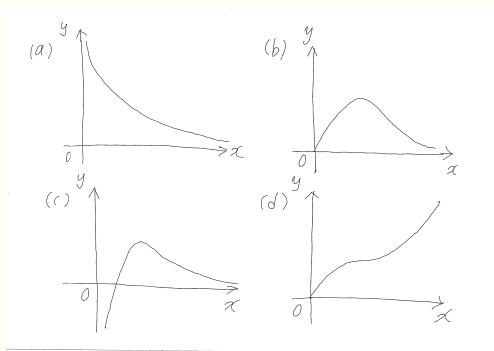
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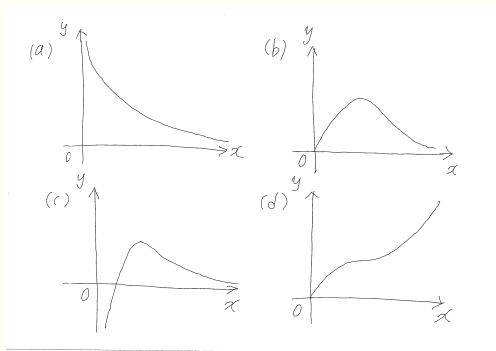
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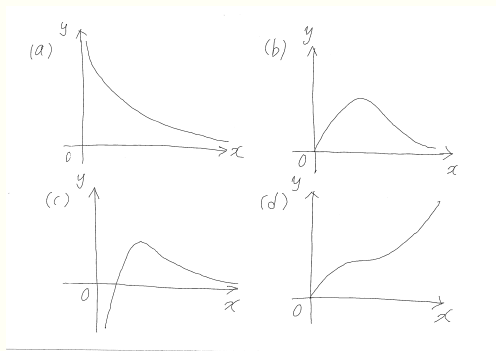
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Answer: **(c)**

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Answer: **(d)**

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Answer: **(a)**

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(c) $\cosh(ix) = \cos(x)$; (d) $\sinh(ix) = \sin(x)$.

(a), (b) and (c) are correct.

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Answer: **(d)**

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Answer: **(d)**

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Answer: **(b)**

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Answer: **(c)**

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Answer: **(d)**

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We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

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Answer: **(a)**

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Answer: **(c)**

How did you do?

Please return answer sheets to the person who wrote them.

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How good was your score?

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5. 11 or 12 : EXCELLENT!!

That's it!

See you in week 8.