

MAS152: Essential Mathematical Skills & Techniques

Prof Elizabeth Winstanley
mas-engineering@sheffield.ac.uk

Monday 10th February 2020, 2pm
Arts Tower LT4

This semester

This semester will proceed as the last one: videos/tests released twice a week, each set followed by a problem class.

This semester will proceed as the last one: videos/tests released twice a week, each set followed by a problem class.

The deadlines for the videos/tests will be slightly different:

This semester will proceed as the last one: videos/tests released twice a week, each set followed by a problem class.

The deadlines for the videos/tests will be slightly different:

- videos released on Monday at 5pm, due Thursday at 11am;

This semester will proceed as the last one: videos/tests released twice a week, each set followed by a problem class.

The deadlines for the videos/tests will be slightly different:

- videos released on Monday at 5pm, due Thursday at 11am;
- videos released on Thursday at 9am, due Monday at 5pm.

This semester will proceed as the last one: videos/tests released twice a week, each set followed by a problem class.

The deadlines for the videos/tests will be slightly different:

- videos released on Monday at 5pm, due Thursday at 11am;
- videos released on Thursday at 9am, due Monday at 5pm.

Some tutors may have changed. We hope this isn't too much of a disappointment!

There will be one other full-class lecture:

There will be one other full-class lecture:

- Monday 30th March (week 8), 2pm, Alfred Denny LT2

There will be one other full-class lecture:

- Monday 30th March (week 8), 2pm, Alfred Denny LT2

That lecture will cover exam technique.

Discussion board

Discussion board

Please remember the discussion board (found via the course webpage) which is by far the best place to ask questions.

Class test

What follows is a test on Semester 1 material.

What follows is a test on Semester 1 material. I will not be collecting marks.

What follows is a test on Semester 1 material. I will not be collecting marks. The test is purely for you to see how you can manage under test conditions.

What follows is a test on Semester 1 material. I will not be collecting marks. The test is purely for you to see how you can manage under test conditions.

Calculators are allowed, but not strictly necessary.

What follows is a test on Semester 1 material. I will not be collecting marks. The test is purely for you to see how you can manage under test conditions.

Calculators are allowed, but not strictly necessary. Notes from last semester and smartphones should be put away!

The test is multiple choice.

The test is multiple choice. On one piece of paper, please put your name and the numbers 1–12 to record your answers.

The test is multiple choice. On one piece of paper, please put your name and the numbers 1–12 to record your answers.

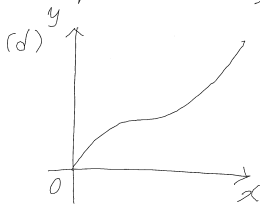
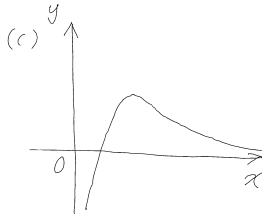
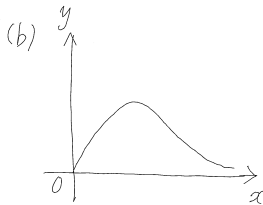
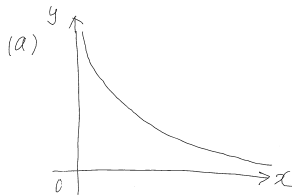
Please do rough workings on another sheet of paper.

The test is multiple choice. On one piece of paper, please put your name and the numbers 1–12 to record your answers.

Please do rough workings on another sheet of paper. At the end we will swap answer sheets and go through the questions.

Question 1

Which sketch represents the function $y = f(x) = \frac{\ln x}{x}$ best ?



Question 2

The first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$ are

(a) $1 - x + \frac{x^2}{2} - \frac{x^3}{2} + \dots$ (b) $1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} + \dots$

(c) $1 - x - \frac{x^2}{2} - \frac{x^3}{2} + \dots$ (d) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16} + \dots$

Question 3

The complex number $\frac{7 + 9i}{1 - 3i}$ is equal to

(a) $\frac{17 - 6i}{5}$; (b) $\frac{-10 - 6i}{5}$;

(c) $\frac{17 + 15i}{5}$; (d) $-2 + 3i$.

Question 4

The complex number $z = -2 - i$ expressed in exponential form $re^{i\theta}$, with θ expressed in radians, is equal to

(a) $\sqrt{5}e^{-2.68i}$; (b) $5e^{-2.68i}$;

(c) $\sqrt{5}e^{0.46i}$; (d) $5e^{0.46i}$.

Question 5

Which one of the following identities are *not* correct ?

(a) $e^{ix} = \cos x + i \sin x$; (b) $e^x = \cosh x + \sinh x$;

(c) $\cosh(ix) = \cos(x)$; (d) $\sinh(ix) = \sin(x)$.

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then $\frac{dy}{dx}$ is

(a) $\frac{1 + 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$;

(b) $\frac{1 - 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$;

(c) $\frac{-1 + 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$;

(d) $\frac{-1 - 2x^2 \sin(2x)}{x + x^2 \cos(2x)}$.

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, then $\frac{dy}{dx}$ is

(a) $\frac{t-1}{t(e^{2t}+1)}$; (b) $\frac{t-1}{t(2e^{2t}+1)}$;

(c) $\frac{2(t-1)}{t(e^{2t}+t^2)}$; (d) $\frac{t(e^{2t}+1)}{t-1}$.

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, the first-order partial derivatives are

$$(a) \quad \frac{\partial f}{\partial x} = 4x, \quad \frac{\partial f}{\partial y} = -9y^2;$$

$$(b) \quad \frac{\partial f}{\partial x} = 3x^2y + 2x^2, \quad \frac{\partial f}{\partial y} = x^3 - 3y^3;$$

$$(c) \quad \frac{\partial f}{\partial x} = 3x^2y + 4x, \quad \frac{\partial f}{\partial y} = x^3 - 9y^2;$$

$$(d) \quad \frac{\partial f}{\partial x} = 3x^2y - 9y^2, \quad \frac{\partial f}{\partial y} = x^3 + 4x.$$

Question 9

The limit $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$ is

(a) -1 ; (b) 1 ;

(c) ∞ ; (d) 0 .

Question 10

When $x = 2$, the function $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ has

- (a) a local maximum; (b) a local minimum;
- (c) a point of inflexion; (d) a non-zero gradient.

Question 11

The value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$ is

(a) -1 ; (b) 1 ;

(c) 5 ; (d) -5 .

Question 12

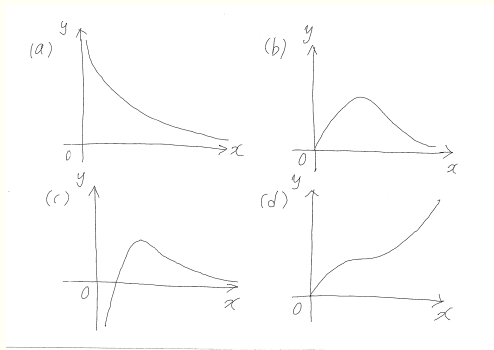
If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, then $\mathbf{a} \times \mathbf{b}$ is

(a) $(-1, -5, -7)$; (b) $(1, -5, 7)$;

(c) $(1, 5, 7)$; (d) -3 .

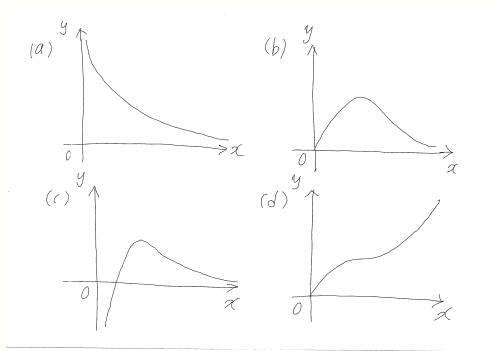
Answers

Question 1



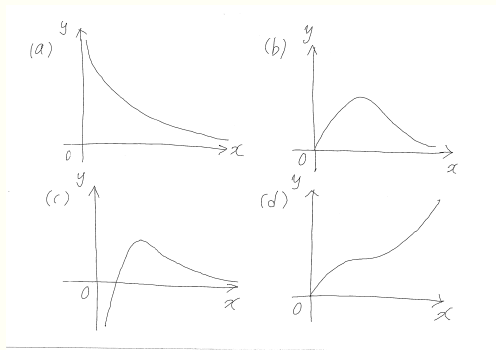
We had $f(x) = \frac{\ln x}{x}$.

Question 1



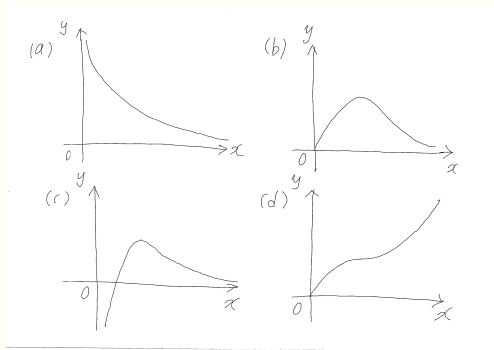
We had $f(x) = \frac{\ln x}{x}$. $f(1) = 0$.

Question 1



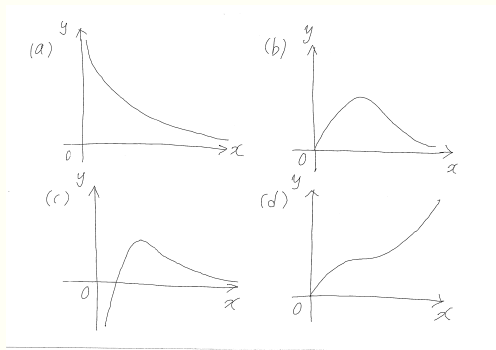
We had $f(x) = \frac{\ln x}{x}$. $f(1) = 0$. For $0 < x < 1$, $f(x) < 0$.

Question 1



We had $f(x) = \frac{\ln x}{x}$. $f(1) = 0$. For $0 < x < 1$, $f(x) < 0$. For large x , $f(x)$ is small because $\ln x \ll x$.

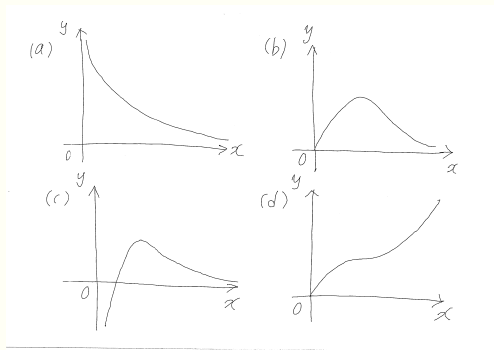
Question 1



We had $f(x) = \frac{\ln x}{x}$. $f(1) = 0$. For $0 < x < 1$, $f(x) < 0$. For large x , $f(x)$ is small because $\ln x \ll x$.

Answer:

Question 1



We had $f(x) = \frac{\ln x}{x}$. $f(1) = 0$. For $0 < x < 1$, $f(x) < 0$. For large x , $f(x)$ is small because $\ln x \ll x$.

Answer: **(c)**

Question 2

Find the first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$.

Question 2

Find the first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$.

Using the binomial theorem,

Question 2

Find the first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$.

Using the binomial theorem,

$$(1 + y)^k = 1 + ky + \frac{k(k-1)}{2!}y^2 + \frac{k(k-1)(k-2)}{3!}y^3 + \dots$$

Question 2

Find the first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$.

Using the binomial theorem,

$$(1 + y)^k = 1 + ky + \frac{k(k-1)}{2!}y^2 + \frac{k(k-1)(k-2)}{3!}y^3 + \dots$$

so

$$(1 + (-2x))^{\frac{1}{2}} =$$

Question 2

Find the first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$.

Using the binomial theorem,

$$(1 + y)^k = 1 + ky + \frac{k(k-1)}{2!}y^2 + \frac{k(k-1)(k-2)}{3!}y^3 + \dots$$

so

$$(1 + (-2x))^{1/2} = 1 - \frac{2}{2}x - \frac{4}{8}x^2 - \frac{24}{48}x^3 + \dots$$

Question 2

Find the first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$.

Using the binomial theorem,

$$(1 + y)^k = 1 + ky + \frac{k(k-1)}{2!}y^2 + \frac{k(k-1)(k-2)}{3!}y^3 + \dots$$

so

$$\begin{aligned}(1 + (-2x))^{1/2} &= 1 - \frac{2}{2}x - \frac{4}{8}x^2 - \frac{24}{48}x^3 + \dots \\ &= 1 - x - \frac{x^2}{2} - \frac{x^3}{2} + \dots\end{aligned}$$

Question 2

Find the first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$.

Using the binomial theorem,

$$(1 + y)^k = 1 + ky + \frac{k(k-1)}{2!}y^2 + \frac{k(k-1)(k-2)}{3!}y^3 + \dots$$

so

$$\begin{aligned}(1 + (-2x))^{\frac{1}{2}} &= 1 - \frac{2}{2}x - \frac{4}{8}x^2 - \frac{24}{48}x^3 + \dots \\ &= 1 - x - \frac{x^2}{2} - \frac{x^3}{2} + \dots\end{aligned}$$

Answer:

Question 2

Find the first four terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$.

Using the binomial theorem,

$$(1 + y)^k = 1 + ky + \frac{k(k-1)}{2!}y^2 + \frac{k(k-1)(k-2)}{3!}y^3 + \dots$$

so

$$\begin{aligned}(1 + (-2x))^{\frac{1}{2}} &= 1 - \frac{2}{2}x - \frac{4}{8}x^2 - \frac{24}{48}x^3 + \dots \\ &= 1 - x - \frac{x^2}{2} - \frac{x^3}{2} + \dots\end{aligned}$$

Answer: **(c)**

Question 3

To simplify the complex number $\frac{7 + 9i}{1 - 3i}$, multiply top and bottom by the conjugate of $1 - 3i$:

Question 3

To simplify the complex number $\frac{7 + 9i}{1 - 3i}$, multiply top and bottom by the conjugate of $1 - 3i$:

$$\frac{7 + 9i}{1 - 3i} =$$

Question 3

To simplify the complex number $\frac{7 + 9i}{1 - 3i}$, multiply top and bottom by the conjugate of $1 - 3i$:

$$\frac{7 + 9i}{1 - 3i} = \left(\frac{7 + 9i}{1 - 3i} \right) \left(\frac{1 + 3i}{1 + 3i} \right)$$

Question 3

To simplify the complex number $\frac{7 + 9i}{1 - 3i}$, multiply top and bottom by the conjugate of $1 - 3i$:

$$\begin{aligned}\frac{7 + 9i}{1 - 3i} &= \left(\frac{7 + 9i}{1 - 3i}\right) \left(\frac{1 + 3i}{1 + 3i}\right) \\ &= \frac{7 + 27i^2 + 9i + 21i}{1^2 + 3^2}\end{aligned}$$

Question 3

To simplify the complex number $\frac{7 + 9i}{1 - 3i}$, multiply top and bottom by the conjugate of $1 - 3i$:

$$\begin{aligned}\frac{7 + 9i}{1 - 3i} &= \left(\frac{7 + 9i}{1 - 3i}\right) \left(\frac{1 + 3i}{1 + 3i}\right) \\ &= \frac{7 + 27i^2 + 9i + 21i}{1^2 + 3^2} \\ &= \frac{-20 + 30i}{10}\end{aligned}$$

Question 3

To simplify the complex number $\frac{7 + 9i}{1 - 3i}$, multiply top and bottom by the conjugate of $1 - 3i$:

$$\begin{aligned}\frac{7 + 9i}{1 - 3i} &= \left(\frac{7 + 9i}{1 - 3i}\right) \left(\frac{1 + 3i}{1 + 3i}\right) \\ &= \frac{7 + 27i^2 + 9i + 21i}{1^2 + 3^2} \\ &= \frac{-20 + 30i}{10} \\ &= -2 + 3i\end{aligned}$$

Question 3

To simplify the complex number $\frac{7 + 9i}{1 - 3i}$, multiply top and bottom by the conjugate of $1 - 3i$:

$$\begin{aligned}\frac{7 + 9i}{1 - 3i} &= \left(\frac{7 + 9i}{1 - 3i}\right) \left(\frac{1 + 3i}{1 + 3i}\right) \\ &= \frac{7 + 27i^2 + 9i + 21i}{1^2 + 3^2} \\ &= \frac{-20 + 30i}{10} \\ &= -2 + 3i\end{aligned}$$

Answer:

Question 3

To simplify the complex number $\frac{7 + 9i}{1 - 3i}$, multiply top and bottom by the conjugate of $1 - 3i$:

$$\begin{aligned}\frac{7 + 9i}{1 - 3i} &= \left(\frac{7 + 9i}{1 - 3i}\right) \left(\frac{1 + 3i}{1 + 3i}\right) \\ &= \frac{7 + 27i^2 + 9i + 21i}{1^2 + 3^2} \\ &= \frac{-20 + 30i}{10} \\ &= -2 + 3i\end{aligned}$$

Answer: **(d)**

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| =$

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} =$

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

The angle that z makes with the negative x -axis is $\alpha =$

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

The angle that z makes with the negative x -axis is $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$.

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

The angle that z makes with the negative x -axis is

$$\alpha = \tan^{-1}\left(\frac{1}{2}\right).$$

Hence, the argument is

$$\theta = \arg(z) =$$

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

The angle that z makes with the negative x -axis is $\alpha = \tan^{-1}(\frac{1}{2})$.

Hence, the argument is

$$\theta = \arg(z) = -(\pi - \tan^{-1}(\frac{1}{2}))$$

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

The angle that z makes with the negative x -axis is $\alpha = \tan^{-1}(\frac{1}{2})$.

Hence, the argument is

$$\theta = \arg(z) = -(\pi - \tan^{-1}(\frac{1}{2})) \approx -2.68.$$

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

The angle that z makes with the negative x -axis is $\alpha = \tan^{-1}(\frac{1}{2})$.

Hence, the argument is

$$\theta = \arg(z) = -(\pi - \tan^{-1}(\frac{1}{2})) \approx -2.68.$$

Hence $z = \sqrt{5}e^{-2.68i}$.

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

The angle that z makes with the negative x -axis is $\alpha = \tan^{-1}(\frac{1}{2})$.

Hence, the argument is

$$\theta = \arg(z) = -(\pi - \tan^{-1}(\frac{1}{2})) \approx -2.68.$$

Hence $z = \sqrt{5}e^{-2.68i}$.

Answer:

Question 4

Express the complex number $z = -2 - i$ in exponential form $re^{i\theta}$, with θ expressed in radians.

Where does z appear on the Argand diagram?

The modulus is $r = |z| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$.

The angle that z makes with the negative x -axis is $\alpha = \tan^{-1}(\frac{1}{2})$.

Hence, the argument is

$$\theta = \arg(z) = -(\pi - \tan^{-1}(\frac{1}{2})) \approx -2.68.$$

Hence $z = \sqrt{5}e^{-2.68i}$.

Answer: **(a)**

Question 5

(a) $e^{ix} = \cos x + i \sin x$; (b) $e^x = \cosh x + \sinh x$;

(c) $\cosh(ix) = \cos(x)$; (d) $\sinh(ix) = \sin(x)$.

(a), (b) and (c) are correct.

Question 5

(a) $e^{ix} = \cos x + i \sin x$; (b) $e^x = \cosh x + \sinh x$;

(c) $\cosh(ix) = \cos(x)$; (d) $\sinh(ix) = \sin(x)$.

(a), (b) and (c) are correct.

(d) would be correct if the right-hand side was multiplied by i .

Answer:

Question 5

(a) $e^{ix} = \cos x + i \sin x$; (b) $e^x = \cosh x + \sinh x$;

(c) $\cosh(ix) = \cos(x)$; (d) $\sinh(ix) = \sin(x)$.

(a), (b) and (c) are correct.

(d) would be correct if the right-hand side was multiplied by i .

Answer: **(d)**

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then find $\frac{dy}{dx}$.

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then find $\frac{dy}{dx}$.

Using the chain rule $\frac{dy}{dx} = \left(\frac{dy}{du}\right) \cdot \left(\frac{du}{dx}\right)$,

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then find $\frac{dy}{dx}$.

Using the chain rule $\frac{dy}{dx} = \left(\frac{dy}{du}\right) \cdot \left(\frac{du}{dx}\right)$,

$$\frac{dy}{dx} = \left(\frac{1}{\frac{1}{x} + \cos(2x)}\right) \left(-\frac{1}{x^2} - 2\sin(2x)\right)$$

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then find $\frac{dy}{dx}$.

Using the chain rule $\frac{dy}{dx} = \left(\frac{dy}{du}\right) \cdot \left(\frac{du}{dx}\right)$,

$$\begin{aligned}\frac{dy}{dx} &= \left(\frac{1}{\frac{1}{x} + \cos(2x)}\right) \left(-\frac{1}{x^2} - 2\sin(2x)\right) \\ &= \left(\frac{x}{1 + x\cos(2x)}\right) \left(\frac{-1 - 2x^2\sin(2x)}{x^2}\right)\end{aligned}$$

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then find $\frac{dy}{dx}$.

Using the chain rule $\frac{dy}{dx} = \left(\frac{dy}{du}\right) \cdot \left(\frac{du}{dx}\right)$,

$$\begin{aligned}\frac{dy}{dx} &= \left(\frac{1}{\frac{1}{x} + \cos(2x)}\right) \left(-\frac{1}{x^2} - 2\sin(2x)\right) \\ &= \left(\frac{x}{1 + x\cos(2x)}\right) \left(\frac{-1 - 2x^2\sin(2x)}{x^2}\right) \\ &= \frac{-1 - 2x^2\sin(2x)}{x + x^2\cos(2x)}.\end{aligned}$$

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then find $\frac{dy}{dx}$.

Using the chain rule $\frac{dy}{dx} = \left(\frac{dy}{du}\right) \cdot \left(\frac{du}{dx}\right)$,

$$\begin{aligned}\frac{dy}{dx} &= \left(\frac{1}{\frac{1}{x} + \cos(2x)}\right) \left(-\frac{1}{x^2} - 2\sin(2x)\right) \\ &= \left(\frac{x}{1 + x\cos(2x)}\right) \left(\frac{-1 - 2x^2\sin(2x)}{x^2}\right) \\ &= \frac{-1 - 2x^2\sin(2x)}{x + x^2\cos(2x)}.\end{aligned}$$

Answer:

Question 6

If $y = \ln\left(\frac{1}{x} + \cos(2x)\right)$ then find $\frac{dy}{dx}$.

Using the chain rule $\frac{dy}{dx} = \left(\frac{dy}{du}\right) \cdot \left(\frac{du}{dx}\right)$,

$$\begin{aligned}\frac{dy}{dx} &= \left(\frac{1}{\frac{1}{x} + \cos(2x)}\right) \left(-\frac{1}{x^2} - 2\sin(2x)\right) \\ &= \left(\frac{x}{1 + x\cos(2x)}\right) \left(\frac{-1 - 2x^2\sin(2x)}{x^2}\right) \\ &= \frac{-1 - 2x^2\sin(2x)}{x + x^2\cos(2x)}.\end{aligned}$$

Answer: **(d)**

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$
and $y = t - \ln t$, find $\frac{dy}{dx}$.

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$
and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} =$

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$
and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} = 2e^{2t} + 1$

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$
and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} = 2e^{2t} + 1$ and $\frac{dy}{dt} =$

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} = 2e^{2t} + 1$ and $\frac{dy}{dt} = 1 - 1/t$.

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} = 2e^{2t} + 1$ and $\frac{dy}{dt} = 1 - 1/t$.

Hence $\frac{dy}{dx} =$

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} = 2e^{2t} + 1$ and $\frac{dy}{dt} = 1 - 1/t$.

Hence $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} = 2e^{2t} + 1$ and $\frac{dy}{dt} = 1 - 1/t$.

Hence $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-1/t}{2e^{2t}+1}$

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} = 2e^{2t} + 1$ and $\frac{dy}{dt} = 1 - 1/t$.

Hence $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-1/t}{2e^{2t}+1} = \frac{t-1}{t(2e^{2t}+1)}$.

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} = 2e^{2t} + 1$ and $\frac{dy}{dt} = 1 - 1/t$.

Hence $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-1/t}{2e^{2t}+1} = \frac{t-1}{t(2e^{2t}+1)}$.

Answer:

Question 7

If x and y are given in terms of a parameter t by $x = e^{2t} + t$ and $y = t - \ln t$, find $\frac{dy}{dx}$.

We have $\frac{dx}{dt} = 2e^{2t} + 1$ and $\frac{dy}{dt} = 1 - 1/t$.

Hence $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1-1/t}{2e^{2t}+1} = \frac{t-1}{t(2e^{2t}+1)}$.

Answer: **(b)**

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, find the first-order partial derivatives.

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, find the first-order partial derivatives.

$$\frac{\partial f}{\partial x} =$$

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, find the first-order partial derivatives.

$$\frac{\partial f}{\partial x} = 3x^2y$$

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, find the first-order partial derivatives.

$$\frac{\partial f}{\partial x} = 3x^2y + 4x;$$

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, find the first-order partial derivatives.

$$\begin{aligned}\frac{\partial f}{\partial x} &= 3x^2y + 4x; \\ \frac{\partial f}{\partial y} &= \end{aligned}$$

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, find the first-order partial derivatives.

$$\frac{\partial f}{\partial x} = 3x^2y + 4x;$$

$$\frac{\partial f}{\partial y} = x^3$$

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, find the first-order partial derivatives.

$$\frac{\partial f}{\partial x} = 3x^2y + 4x;$$

$$\frac{\partial f}{\partial y} = x^3 - 9y^2.$$

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, find the first-order partial derivatives.

$$\frac{\partial f}{\partial x} = 3x^2y + 4x;$$
$$\frac{\partial f}{\partial y} = x^3 - 9y^2.$$

Answer:

Question 8

Given a function of two variables, $f(x, y) = x^3y - 3y^3 + 2x^2$, find the first-order partial derivatives.

$$\frac{\partial f}{\partial x} = 3x^2y + 4x;$$
$$\frac{\partial f}{\partial y} = x^3 - 9y^2.$$

Answer: **(c)**

Question 9

Find the limit $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$.

Question 9

Find the limit $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$.

Using l'Hôpital's rule,

Question 9

Find the limit $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$.

Using l'Hôpital's rule,

$$\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$$

Question 9

Find the limit $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$.

Using l'Hôpital's rule,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{f(x)}{g(x)} &= \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin(x) \cos(x)}{2x - 1} \end{aligned}$$

Question 9

Find the limit $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$.

Using l'Hôpital's rule,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{f(x)}{g(x)} &= \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin(x) \cos(x)}{2x - 1} \\ &= 0 \end{aligned}$$

Question 9

Find the limit $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$.

Using l'Hôpital's rule,

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} &= \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin(x) \cos(x)}{2x - 1} \\ &= 0\end{aligned}$$

Answer:

Question 9

Find the limit $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2 - x}$.

Using l'Hôpital's rule,

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} &= \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin(x) \cos(x)}{2x - 1} \\ &= 0\end{aligned}$$

Answer: **(d)**

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) =$$

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) = 4x^3 - 24x^2 + 46x - 28$$

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$\begin{aligned} f'(x) &= 4x^3 - 24x^2 + 46x - 28 \\ \text{so } f'(2) &= 32 - 96 + 92 - 28 \end{aligned}$$

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$\begin{aligned}f'(x) &= 4x^3 - 24x^2 + 46x - 28 \\ \text{so } f'(2) &= 32 - 96 + 92 - 28 = 0.\end{aligned}$$

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) = 4x^3 - 24x^2 + 46x - 28$$

$$\text{so } f'(2) = 32 - 96 + 92 - 28 = 0.$$

$$f''(x) =$$

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) = 4x^3 - 24x^2 + 46x - 28$$

$$\text{so } f'(2) = 32 - 96 + 92 - 28 = 0.$$

$$f''(x) = 12x^2 - 48x + 46$$

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) = 4x^3 - 24x^2 + 46x - 28$$

$$\text{so } f'(2) = 32 - 96 + 92 - 28 = 0.$$

$$f''(x) = 12x^2 - 48x + 46$$

$$\text{so } f''(2) = 48 - 96 + 46$$

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) = 4x^3 - 24x^2 + 46x - 28$$

$$\text{so } f'(2) = 32 - 96 + 92 - 28 = 0.$$

$$f''(x) = 12x^2 - 48x + 46$$

$$\text{so } f''(2) = 48 - 96 + 46 = -2$$

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) = 4x^3 - 24x^2 + 46x - 28$$

$$\text{so } f'(2) = 32 - 96 + 92 - 28 = 0.$$

$$f''(x) = 12x^2 - 48x + 46$$

$$\text{so } f''(2) = 48 - 96 + 46 = -2 < 0.$$

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) = 4x^3 - 24x^2 + 46x - 28$$

$$\text{so } f'(2) = 32 - 96 + 92 - 28 = 0.$$

$$f''(x) = 12x^2 - 48x + 46$$

$$\text{so } f''(2) = 48 - 96 + 46 = -2 < 0.$$

Thus f has a local maximum.

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) = 4x^3 - 24x^2 + 46x - 28$$

$$\text{so } f'(2) = 32 - 96 + 92 - 28 = 0.$$

$$f''(x) = 12x^2 - 48x + 46$$

$$\text{so } f''(2) = 48 - 96 + 46 = -2 < 0.$$

Thus f has a local maximum.

Answer:

Question 10

We look at the first and second derivatives of $f(x) = x^4 - 8x^3 + 23x^2 - 28x + 2$ at $x = 2$.

$$f'(x) = 4x^3 - 24x^2 + 46x - 28$$

$$\text{so } f'(2) = 32 - 96 + 92 - 28 = 0.$$

$$f''(x) = 12x^2 - 48x + 46$$

$$\text{so } f''(2) = 48 - 96 + 46 = -2 < 0.$$

Thus f has a local maximum.

Answer: **(a)**

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Now, $\mathbf{a} \cdot \mathbf{b} =$

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Now, $\mathbf{a} \cdot \mathbf{b} = 3 \cdot 1 +$

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Now, $\mathbf{a} \cdot \mathbf{b} = 3 \cdot 1 + (-1) \cdot \mu +$

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Now, $\mathbf{a} \cdot \mathbf{b} = 3 \cdot 1 + (-1) \cdot \mu + 2 \cdot (-1)$

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Now, $\mathbf{a} \cdot \mathbf{b} = 3 \cdot 1 + (-1) \cdot \mu + 2 \cdot (-1) = 1 - \mu$.

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Now, $\mathbf{a} \cdot \mathbf{b} = 3 \cdot 1 + (-1) \cdot \mu + 2 \cdot (-1) = 1 - \mu$.

Hence \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mu = 1$.

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Now, $\mathbf{a} \cdot \mathbf{b} = 3 \cdot 1 + (-1) \cdot \mu + 2 \cdot (-1) = 1 - \mu$.

Hence \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mu = 1$.

Answer:

Question 11

Find the value of μ for which the vector $\mathbf{a} = (3, -1, 2)$ is perpendicular to the vector $\mathbf{b} = (1, \mu, -1)$.

Vectors \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

Now, $\mathbf{a} \cdot \mathbf{b} = 3 \cdot 1 + (-1) \cdot \mu + 2 \cdot (-1) = 1 - \mu$.

Hence \mathbf{a} and \mathbf{b} are perpendicular if and only if $\mu = 1$.

Answer: **(b)**

Question 12

If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, find $\mathbf{a} \times \mathbf{b}$.

$$\mathbf{a} \times \mathbf{b} =$$

Question 12

If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, find $\mathbf{a} \times \mathbf{b}$.

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

Question 12

If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, find $\mathbf{a} \times \mathbf{b}$.

$$\begin{aligned}\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 2 \\ 2 & 1 & -1 \end{vmatrix} \\ &= \mathbf{i}(3 - 2) - \mathbf{j}(-1 - 4) + \mathbf{k}(1 + 6)\end{aligned}$$

Question 12

If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, find $\mathbf{a} \times \mathbf{b}$.

$$\begin{aligned}\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 2 \\ 2 & 1 & -1 \end{vmatrix} \\ &= \mathbf{i}(3 - 2) - \mathbf{j}(-1 - 4) + \mathbf{k}(1 + 6) \\ &= \mathbf{i} + 5\mathbf{j} + 7\mathbf{k}.\end{aligned}$$

Question 12

If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, find $\mathbf{a} \times \mathbf{b}$.

$$\begin{aligned}\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 2 \\ 2 & 1 & -1 \end{vmatrix} \\ &= \mathbf{i}(3 - 2) - \mathbf{j}(-1 - 4) + \mathbf{k}(1 + 6) \\ &= \mathbf{i} + 5\mathbf{j} + 7\mathbf{k}.\end{aligned}$$

Answer:

Question 12

If $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (2, 1, -1)$, find $\mathbf{a} \times \mathbf{b}$.

$$\begin{aligned}\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 2 \\ 2 & 1 & -1 \end{vmatrix} \\ &= \mathbf{i}(3 - 2) - \mathbf{j}(-1 - 4) + \mathbf{k}(1 + 6) \\ &= \mathbf{i} + 5\mathbf{j} + 7\mathbf{k}.\end{aligned}$$

Answer: **(c)**

How did you do?

Please return answer sheets to the person who wrote them.

Please return answer sheets to the person who wrote them.

How good was your score?

Please return answer sheets to the person who wrote them.

How good was your score?

1. 0–2: A lot of work needed!

Please return answer sheets to the person who wrote them.

How good was your score?

1. 0–2: A lot of work needed!
2. 3–5: Something like a pass.

Please return answer sheets to the person who wrote them.

How good was your score?

1. 0–2: A lot of work needed!
2. 3–5: Something like a pass.
3. 6–8: A respectable score.

Please return answer sheets to the person who wrote them.

How good was your score?

1. 0–2: A lot of work needed!
2. 3–5: Something like a pass.
3. 6–8: A respectable score.
4. 9 or 10: Very well done!

Please return answer sheets to the person who wrote them.

How good was your score?

1. 0–2: A lot of work needed!
2. 3–5: Something like a pass.
3. 6–8: A respectable score.
4. 9 or 10: Very well done!
5. 11 or 12 : EXCELLENT!!

That's it!

See you in week 8.