

MAS152: Essential Mathematical Skills & Techniques

Dr James Cranch

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Monday 17th October 2016, 2pm
St George's Church

Course matters

Online tests

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Watch each video to the end to find the link to the tests.

Online materials

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Your comments

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Similarly, please do click the *thumbs up* or *thumbs down* buttons on Youtube if you particularly like or dislike a video as it will help us improve the materials.

Reading week

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Complex numbers

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Why imaginary numbers?

We know that

$$x^2 \geq 0$$

for all x in the real numbers \mathbb{R} .

Consider the following algebraic equation

$$x^2 = -1$$

which has no solutions (roots) in \mathbb{R} .

Define i , the *imaginary unit*, to be a solution of the equation $i^2 = -1$. In other words,

$$i = \sqrt{-1}.$$

A complex number

$$z = x + iy, \quad x, y, \in \mathbb{R}$$

has two parts

$$x = \Re(z), \quad y = \Im(z),$$

the *real* and *imaginary* parts, respectively.

Complex algebra

Two complex numbers

$$z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2$$

are identical (that is, $z_1 = z_2$) if and only if

$$x_1 = x_2 \quad \text{and} \quad y_1 = y_2.$$

Addition, subtraction & multiplication

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

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These can be verified by computations. e.g. the third one

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$$\begin{aligned} z_1 z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1 x_2 + i^2 y_1 y_2 + ix_1 y_2 + iy_1 x_2 \end{aligned}$$

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Complex conjugate

For $z = x + iy$, we define its *conjugate* by

$$\bar{z} = x - iy.$$

Then

$$z\bar{z} = (x + iy)(x - iy) = x^2 + y^2 \in \mathbb{R}, \geq 0$$

Complex division

The trick of *realising the denominator* works as follows:

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In the general case,

$$\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)} = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}.$$

Some rules

Commutative laws

$$z_1 + z_2 = z_2 + z_1,$$

$$z_1 z_2 = z_2 z_1;$$

Associative laws

$$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3,$$

$$z_1(z_2 z_3) = (z_1 z_2)z_3;$$

Distributive laws

$$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3.$$

These can be checked by direct computations.

More on conjugates

For $z = x + iy$, $\bar{z} = x - iy$,

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Hence

$$\Re(z) = x = \frac{1}{2}(z + \bar{z}),$$

$$\Im(z) = y = \frac{1}{2i}(z - \bar{z}).$$

Rules about the conjugate

For $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, we have

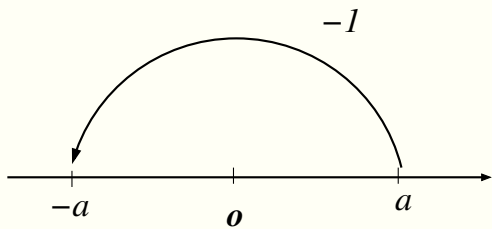
$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2},$$

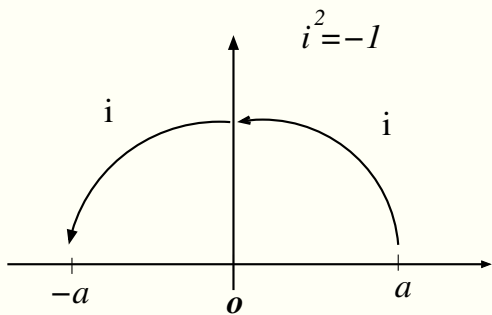
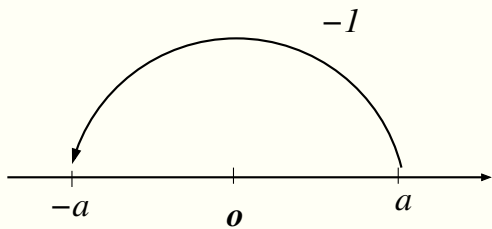
$$\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2},$$

$$\overline{z_1 z_2} = \overline{z_1} \overline{z_2},$$

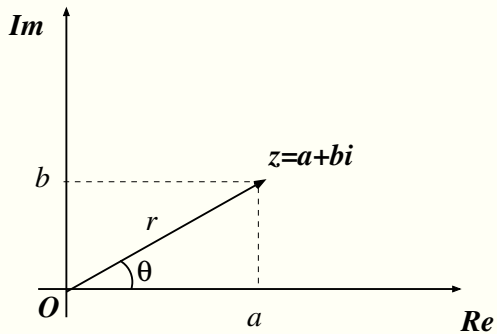
$$\overline{\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}} = \begin{pmatrix} \overline{z_1} \\ \overline{z_2} \end{pmatrix}.$$

Argand diagram





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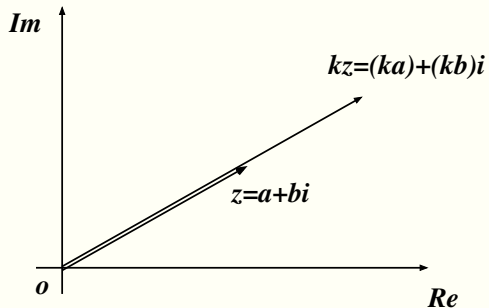
For $z = a + bi$,

- The *modulus* is $r = |z|$,
- The *argument* is $\theta = \arg(z)$,
- As $r^2 = a^2 + b^2$, we have $|z| = \sqrt{a^2 + b^2}$,
- We have $\tan(\arg(z)) = b/a$,
- $a = r \cos \theta$ and $b = r \sin \theta$.

Different choices of θ are possible, but the *principal argument* is defined by $-\pi < \arg(z) \leq \pi$.

Multiplication by a real number

Extension/compression along the same direction

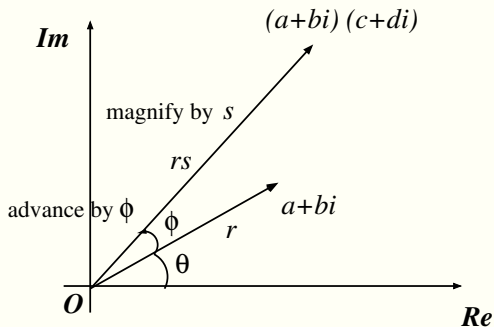


Multiplication by a complex number

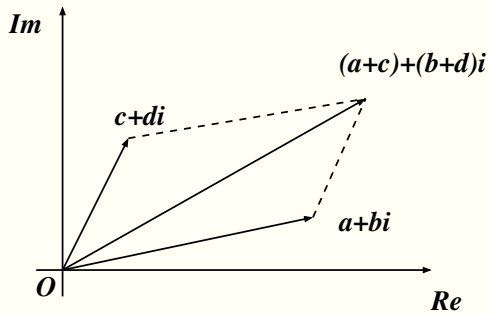
$$a + bi = r (\cos \theta + i \sin \theta)$$

$$c + di = s (\cos \phi + i \sin \phi)$$

$$\begin{aligned}(a + bi)(c + di) &= rs (\cos \theta + i \sin \theta) (\cos \phi + i \sin \phi) \\ &= rs [(\cos \theta \cos \phi - \sin \theta \sin \phi) + i(\cos \theta \sin \phi + \sin \theta \cos \phi)] \\ &= rs (\cos(\theta + \phi) + i \sin(\theta + \phi))\end{aligned}$$



Addition, again



What's it useful for?

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For example, the polynomial $x^2 - 6x + 10 = 0$ has no real roots, but has complex roots $3 + i$ and $3 - i$. We can interpret this as telling us which real number is closest to being a root (namely 3) and also telling us something about how far it is from having a root.

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It turns out that many important examples are governed by equations:

- Positive real roots mean exponential growth;
- Negative real roots mean exponential decay;
- Complex roots mean *oscillations*.

Still to come...

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Reminders:

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- email address mas-engineering@sheffield.ac.uk
- website <http://engmaths.group.shef.ac.uk/mas152>
(also accessible through MOLE).