MAS152: Essential Mathematical Skills & Techniques

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Monday 16th October 2017, 1pm
Diamond LT4

Course matters

Online tests

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Watch each video to the end to find the link to the tests.

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Your comments

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Similarly, please do click the *thumbs up* or *thumbs down* buttons on Youtube if you particularly like or dislike a video as it will help us improve the materials.

Reading week

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Complex numbers

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Why imaginary numbers?

We know that

$$x^2 \geq 0$$

for all x in the real numbers $\mathbb{R}$.

Consider the following algebraic equation

$$x^2 = -1$$

which has no solutions(roots) in $\mathbb{R}$.

Define i, the *imaginary unit*, to be a solution of the equation $i^2 = -1$. In other words,

$$i = \sqrt{-1}.$$

A *complex number*

$$z = x + iy, \qquad x, y, \in \mathbb{R}$$

has two parts

$$x = \Re(z), \qquad y = \Im(z),$$

the *real* and *imaginary* parts, respectively.

Complex algebra

Two complex numbers

$$z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2$$

are identical (that is, $z_1 = z_2$) if and only if

$$x_1 = x_2 \qquad \text{and} \qquad y_1 = y_2.$$

Addition, subtraction & multiplication

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

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These can be verified by computations. e.g. the third one

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Complex conjugate

For $z = x + iy$, we define its *conjugate* by

$$\overline{z} = x - iy.$$

Then

$$z\overline{z} = (x+iy)(x-iy) = x^2 + y^2 \in \mathbb{R}, \ \ \geq 0$$

Complex division

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$$\frac{z_1}{z_2} = \frac{z_1\overline{z_2}}{z_2\overline{z_2}}.$$

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$$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}.$$

This is reminiscent of *rationalisng the denominator*:

$$\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}}\,\frac{2-\sqrt{3}}{2-\sqrt{3}} = 2-\sqrt{3}.$$

Some rules

Commutative laws

$$z_1 + z_2 = z_2 + z_1,$$

$$z_1 z_2 = z_2 z_1;$$

Associative laws

$$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3,$$

$$z_1(z_2 z_3) = (z_1 z_2) z_3;$$

Distributive laws

$$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3.$$

These can be checked by direct computations.

More on conjugates

For $z = x + iy$, $\overline{z} = x - iy$,

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Hence

$$\Re(z) = x = \frac{1}{2}(z + \overline{z}),$$

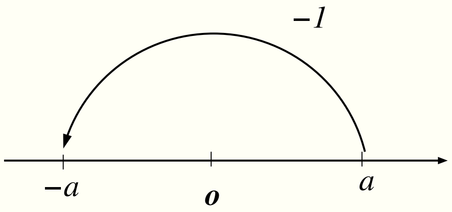
$$\Im(z) = y = \frac{1}{2i}(z - \overline{z}).$$

Rules about the conjugate

For $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, we have

$$\begin{aligned}
\overline{z_1 + z_2} &= \overline{z_1} + \overline{z_2}, \\
\overline{z_1 - z_2} &= \overline{z_1} - \overline{z_2}, \\
\overline{z_1\, z_2} &= \overline{z_1}\,\overline{z_2}, \\
\overline{\left(\frac{z_1}{z_2}\right)} &= \frac{(\overline{z_1})}{(\overline{z_2})}.
\end{aligned}$$

Prelude to **Argand diagram**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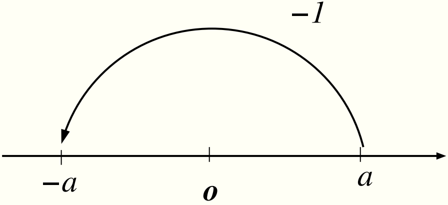

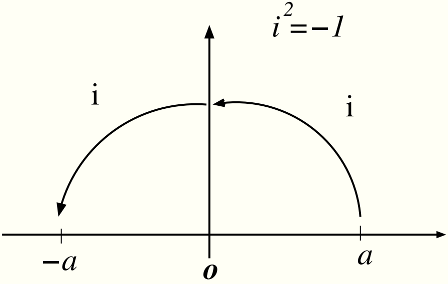

This idea combines complex numbers with planar geometry.
(We will learn the details on videos and at tutorials.)

What's it useful for?

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For example, the polynomial $x^2 - 6x + 10 = 0$ has no real roots, but has complex roots $3 + i$ and $3 - i$.

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For example, the polynomial $x^2 - 6x + 10 = 0$ has no real roots, but has complex roots $3 + i$ and $3 - i$. We can interpret this as telling us which real number is closest to being a root (namely 3) and also telling us something about how far it is from having a root.

Towards the end of the year, we'll also use them to study feedback systems, which occur throughout engineering.

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It turns out that many important examples are governed by equations:

- Positive real roots mean exponential growth;
- Negative real roots mean exponential decay;
- Complex roots mean *oscillations*.

Still to come...

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Reminders:

- No classes in Week 7
- email address `mas-engineering@sheffield.ac.uk`
- website `http://engmaths.group.shef.ac.uk/mas152` (also accessible through MOLE).