MAS152: Essential Mathematical Skills & Techniques

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Monday 8th October 2018, 1pm
Stephenson LT1

Course matters

Online tests

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Reminder: You must watch each video to the end to find the link to the tests.

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Each test is only worth about 0.15% of the total module credit.

Your comments

We are interested to know your opinions about this course:

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Please also click the *thumbs up* or *thumbs down* buttons on Youtube if you particularly like or dislike a video as it will help us improve the materials.

Reading week

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You should use the time to revise or catch up with the material so far, e.g by working on exercises.

Complex numbers

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To help you to get comfortable in their use, we will cover some of the basics today.

Why imaginary numbers?

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Define i, the *imaginary unit*, to be a solution of the equation $i^2 = -1$. In other words,

$$i = \sqrt{-1}.$$

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It has two parts:

$$x = \Re(z), \qquad y = \Im(z),$$

known as the *real* and *imaginary* parts, respectively.

Complex algebra

Two complex numbers

$$z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2$$

are identical (that is, $z_1 = z_2$) if and only if

$$x_1 = x_2 \qquad \text{and} \qquad y_1 = y_2.$$

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Complex conjugate

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Notice that

$$z\overline{z} = (x + iy)(x - iy) = x^2 + y^2 \in \mathbb{R}, \quad \geq 0$$

Complex division

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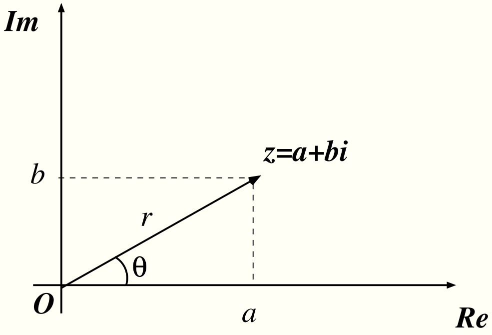
Argand diagram

We can represent complex numbers in a 2-dimensional plane, known as the *Argand diagram*:



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Different choices of θ are possible, but the *principal argument* is defined by $-\pi < \arg(z) \leq \pi$.

What's it useful for?

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For example, the polynomial $x^2 - 6x + 10 = 0$ has no real roots, but has complex roots $3 + i$ and $3 - i$. We can interpret this as telling us which real number is closest to being a root (namely 3) and also telling us something about how far it is from having a root.

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- Complex roots mean *oscillations*.

Still to come...

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Reminders:

- No classes in Week 7
- email address `mas-engineering@sheffield.ac.uk`
- website `http://engmaths.group.shef.ac.uk/mas152` (also accessible through MOLE).