

# MAS151: Civil Engineering Mathematics

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Friday 18th October 2019, 12noon

Hicks Building LT1

# Course matters

# Online tests

If you are still having any problems accessing the online materials, remember that there are some things that can help.

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Reminder: You must watch each video to the end to find the link to the tests.



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**Each test is only worth about 0.15% of the total module credit.**

**Your comments**



We are interested to know your opinions about this course:

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Please also click the *thumbs up* or *thumbs down* buttons on Youtube if you particularly like or dislike a video as it will help us improve the materials.

# Complex numbers

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To help you to get comfortable in their use, we will cover some of the basics today.

**Why imaginary numbers?**



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Define  $i$ , the *imaginary unit*, to be a solution of the equation  $i^2 = -1$ . In other words,

$$i = \sqrt{-1}.$$

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It has two parts:

$$x = \Re(z), \quad y = \Im(z),$$

known as the *real* and *imaginary* parts, respectively.

# Complex algebra

Two complex numbers

$$z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2$$

are identical (that is,  $z_1 = z_2$ ) if and only if

$$x_1 = x_2 \quad \text{and} \quad y_1 = y_2.$$



# **Addition, subtraction & multiplication**

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# Complex conjugate

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Notice that

$$z\bar{z} = (x + iy)(x - iy) = x^2 + y^2 \in \mathbb{R}, \geq 0$$

# Complex division

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**What's it useful for?**

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For example, the polynomial  $x^2 - 6x + 10 = 0$  has no real roots, but has complex roots  $3 + i$  and  $3 - i$ . We can interpret this as telling us which real number is closest to being a root (namely 3) and also telling us something about how far it is from having a root.

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- Complex roots mean *oscillations*.

**Still to come...**

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### Reminders:

- email address [mas-engineering@sheffield.ac.uk](mailto:mas-engineering@sheffield.ac.uk)
- website <http://engmaths.group.shef.ac.uk/mas151>  
(also accessible through MOLE).