

MAS140: Mathematics (Chemical)

Prof Koji Ohkitani

mas-engineering@sheffield.ac.uk

Wednesday 18th October 2017, 1pm
Diamond LT1

Course matters

Online tests

Some people had problems in the early stages with the online tests.

Some people had problems in the early stages with the online tests. I am sorry if you wasted time.

Some people had problems in the early stages with the online tests. I am sorry if you wasted time. There have been many posts on the discussion board about this,

Some people had problems in the early stages with the online tests. I am sorry if you wasted time. There have been many posts on the discussion board about this, so if you still have problems you could see what people have written there.

Some people had problems in the early stages with the online tests. I am sorry if you wasted time. There have been many posts on the discussion board about this, so if you still have problems you could see what people have written there.

There are, however, some things that can help.

Some people had problems in the early stages with the online tests. I am sorry if you wasted time. There have been many posts on the discussion board about this, so if you still have problems you could see what people have written there.

There are, however, some things that can help.

If the videos aren't playing properly, try

- using a different browser (Firefox seems to work most reliably);
- ensuring javascript is enabled;
- clearing your browser's cache;
- pressing refresh.

Some people had problems in the early stages with the online tests. I am sorry if you wasted time. There have been many posts on the discussion board about this, so if you still have problems you could see what people have written there.

There are, however, some things that can help.

If the videos aren't playing properly, try

- using a different browser (Firefox seems to work most reliably);
- ensuring javascript is enabled;
- clearing your browser's cache;
- pressing refresh.

This solves most problems.

Some people had problems in the early stages with the online tests. I am sorry if you wasted time. There have been many posts on the discussion board about this, so if you still have problems you could see what people have written there.

There are, however, some things that can help.

If the videos aren't playing properly, try

- using a different browser (Firefox seems to work most reliably);
- ensuring javascript is enabled;
- clearing your browser's cache;
- pressing refresh.

This solves most problems.

Watch each video to the end to find the link to the tests.

Online materials

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back.

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back. If these aren't enough, start a thread on the discussion board.

Don't forget about the online notes and exercises, found on the course webpage.

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back. If these aren't enough, start a thread on the discussion board.

Don't forget about the online notes and exercises, found on the course webpage.

The notes cover the same material as the videos

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back. If these aren't enough, start a thread on the discussion board.

Don't forget about the online notes and exercises, found on the course webpage.

The notes cover the same material as the videos but not always in precisely the same way.

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back. If these aren't enough, start a thread on the discussion board.

Don't forget about the online notes and exercises, found on the course webpage.

The notes cover the same material as the videos but not always in precisely the same way. We hope that seeing the material presented differently will help you to achieve a thorough understanding.

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back. If these aren't enough, start a thread on the discussion board.

Don't forget about the online notes and exercises, found on the course webpage.

The notes cover the same material as the videos but not always in precisely the same way. We hope that seeing the material presented differently will help you to achieve a thorough understanding.

The exercise sheets have practice questions that are different from the ones in the problem class.

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back. If these aren't enough, start a thread on the discussion board.

Don't forget about the online notes and exercises, found on the course webpage.

The notes cover the same material as the videos but not always in precisely the same way. We hope that seeing the material presented differently will help you to achieve a thorough understanding.

The exercise sheets have practice questions that are different from the ones in the problem class. You can either keep up-to-date as we cover the material

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back. If these aren't enough, start a thread on the discussion board.

Don't forget about the online notes and exercises, found on the course webpage.

The notes cover the same material as the videos but not always in precisely the same way. We hope that seeing the material presented differently will help you to achieve a thorough understanding.

The exercise sheets have practice questions that are different from the ones in the problem class. You can either keep up-to-date as we cover the material or save them for reading week, Christmas and Easter breaks and exam preparation time.

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back. If these aren't enough, start a thread on the discussion board.

Don't forget about the online notes and exercises, found on the course webpage.

The notes cover the same material as the videos but not always in precisely the same way. We hope that seeing the material presented differently will help you to achieve a thorough understanding.

The exercise sheets have practice questions that are different from the ones in the problem class. You can either keep up-to-date as we cover the material or save them for reading week, Christmas and Easter breaks and exam preparation time. Your choice!

Remember that you can find all the sheets from the tutorial classes, with some brief answers on the back. If these aren't enough, start a thread on the discussion board.

Don't forget about the online notes and exercises, found on the course webpage.

The notes cover the same material as the videos but not always in precisely the same way. We hope that seeing the material presented differently will help you to achieve a thorough understanding.

The exercise sheets have practice questions that are different from the ones in the problem class. You can either keep up-to-date as we cover the material or save them for reading week, Christmas and Easter breaks and exam preparation time. Your choice!

Also on the course webpage are the slides from the lecture in
Week 1

Also on the course webpage are the slides from the lecture in Week 1 (and this one!).

Also on the course webpage are the slides from the lecture in Week 1 (and this one!). If you missed it, please read up.

Also on the course webpage are the slides from the lecture in Week 1 (and this one!). If you missed it, please read up.

Reminder: New videos are released at midday on Tuesdays, due in late on Fridays, and midday on Fridays, due in late on Tuesdays.

Also on the course webpage are the slides from the lecture in Week 1 (and this one!). If you missed it, please read up.

Reminder: New videos are released at midday on Tuesdays, due in late on Fridays, and midday on Fridays, due in late on Tuesdays.

If you miss a test the deadline cannot be extended. If you are ill, or unable to do the tests for another good reason, you need to tell us by emailing mas-engineering@sheffield.ac.uk so we can take this into account when working out your total coursework mark.

Also on the course webpage are the slides from the lecture in Week 1 (and this one!). If you missed it, please read up.

Reminder: New videos are released at midday on Tuesdays, due in late on Fridays, and midday on Fridays, due in late on Tuesdays.

If you miss a test the deadline cannot be extended. If you are ill, or unable to do the tests for another good reason, you need to tell us by emailing mas-engineering@sheffield.ac.uk so we can take this into account when working out your total coursework mark. **Remember that each test is only worth about 0.15% of the total module credit.**

Your comments

We are interested to know your opinions about this course via the discussion board,

We are interested to know your opinions about this course via the discussion board, and you will get a thorough questionnaire on all of your courses towards the end of the semester.

We are interested to know your opinions about this course via the discussion board, and you will get a thorough questionnaire on all of your courses towards the end of the semester.

Similarly, please do click the *thumbs up* or *thumbs down* buttons on Youtube if you particularly like or dislike a video as it will help us improve the materials.

Reading week

Week 7 (November 6–10) is a reading week.

Week 7 (November 6–10) is a reading week. There will be no classes or new videos released in this week (the videos released late in week 6 will be due in early in week 8).

Week 7 (November 6–10) is a reading week. There will be no classes or new videos released in this week (the videos released late in week 6 will be due in early in week 8). You should use the time to revise or catch up with the material so far (e.g. by working on exercises).

Complex numbers

In this course we will spend a good amount of time studying
Complex Numbers.

In this course we will spend a good amount of time studying *Complex Numbers*. However, they are so fundamental to engineering mathematics that they may have already appeared elsewhere in your course or could come up before we get to them.

In this course we will spend a good amount of time studying *Complex Numbers*. However, they are so fundamental to engineering mathematics that they may have already appeared elsewhere in your course or could come up before we get to them. To help you to get comfortable in their use, we will cover some of the basics today.

Why imaginary numbers?

We know that

$$x^2 \geq 0$$

for all x in the real numbers \mathbb{R} .

Consider the following algebraic equation

$$x^2 = -1$$

which has no solutions (roots) in \mathbb{R} .

Define i , the *imaginary unit*, to be a solution of the equation $i^2 = -1$. In other words,

$$i = \sqrt{-1}.$$

A complex number

$$z = x + iy, \quad x, y, \in \mathbb{R}$$

has two parts

$$x = \Re(z), \quad y = \Im(z),$$

the *real* and *imaginary* parts, respectively.

Complex algebra

Two complex numbers

$$z_1 = x_1 + iy_1, \quad z_2 = x_2 + iy_2$$

are identical (that is, $z_1 = z_2$) if and only if

$$x_1 = x_2 \quad \text{and} \quad y_1 = y_2.$$

Addition, subtraction & multiplication

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

These can be verified by computations. e.g. the third one

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

These can be verified by computations. e.g. the third one

$$\begin{aligned} z_1 z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1 x_2 + i^2 y_1 y_2 + ix_1 y_2 + iy_1 x_2 \end{aligned}$$

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

These can be verified by computations. e.g. the third one

$$\begin{aligned} z_1 z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1 x_2 + i^2 y_1 y_2 + ix_1 y_2 + iy_1 x_2 \\ &= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1). \end{aligned}$$

Complex conjugate

For $z = x + iy$, we define its *conjugate* by

$$\bar{z} = x - iy.$$

Then

$$z\bar{z} = (x + iy)(x - iy) = x^2 + y^2 \in \mathbb{R}, \geq 0$$

Complex division

The trick of *realising the denominator* works as follows:

$$\frac{z_1}{z_2} = \frac{z_1 \bar{z}_2}{z_2 \bar{z}_2}.$$

In the general case,

The trick of *realising the denominator* works as follows:

$$\frac{z_1}{z_2} = \frac{z_1 \overline{z_2}}{z_2 \overline{z_2}}.$$

In the general case,

$$\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)}$$

The trick of *realising the denominator* works as follows:

$$\frac{z_1}{z_2} = \frac{z_1 \bar{z}_2}{z_2 \bar{z}_2}.$$

In the general case,

$$\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)} = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}.$$

This is reminiscent of *rationalising the denominator*:

$$\frac{1}{2 + \sqrt{3}} = \frac{1}{2 + \sqrt{3}} \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = 2 - \sqrt{3}.$$

Some rules

Commutative laws

$$z_1 + z_2 = z_2 + z_1,$$

$$z_1 z_2 = z_2 z_1;$$

Associative laws

$$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3,$$

$$z_1(z_2 z_3) = (z_1 z_2)z_3;$$

Distributive laws

$$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3.$$

These can be checked by direct computations.

More on conjugates

For $z = x + iy$, $\bar{z} = x - iy$,

$$z + \bar{z} = 2x, \quad z - \bar{z} = 2iy.$$

For $z = x + iy$, $\bar{z} = x - iy$,

$$z + \bar{z} = 2x, \quad z - \bar{z} = 2iy.$$

Hence

$$\Re(z) = x = \frac{1}{2}(z + \bar{z}),$$

$$\Im(z) = y = \frac{1}{2i}(z - \bar{z}).$$

Rules about the conjugate

For $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, we have

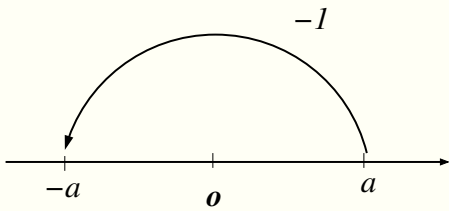
$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2},$$

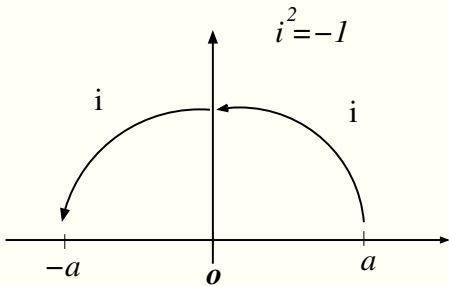
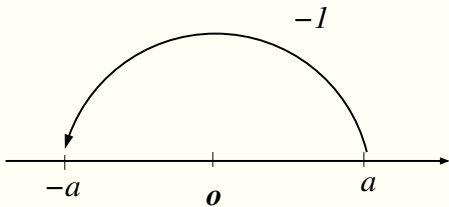
$$\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2},$$

$$\overline{z_1 z_2} = \overline{z_1} \overline{z_2},$$

$$\overline{\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}} = \begin{pmatrix} \overline{z_1} \\ \overline{z_2} \end{pmatrix}.$$

Prelude to **Argand diagram**





This idea combines complex numbers with planar geometry.
(We will learn the details on videos and at tutorials.)

What's it useful for?

When complex numbers were first invented, they were not thought to be of much use. But now they're an important tool.

They give extra algebraic information, even when we only care about real numbers.

When complex numbers were first invented, they were not thought to be of much use. But now they're an important tool.

They give extra algebraic information, even when we only care about real numbers.

For example, the polynomial $x^2 - 6x + 10 = 0$ has no real roots, but has complex roots $3 + i$ and $3 - i$.

When complex numbers were first invented, they were not thought to be of much use. But now they're an important tool.

They give extra algebraic information, even when we only care about real numbers.

For example, the polynomial $x^2 - 6x + 10 = 0$ has no real roots, but has complex roots $3 + i$ and $3 - i$. We can interpret this as telling us which real number is closest to being a root (namely 3) and also telling us something about how far it is from having a root.

Towards the end of the year, we'll also use them to study feedback systems, which occur throughout engineering.

Towards the end of the year, we'll also use them to study feedback systems, which occur throughout engineering.

It turns out that many important examples are governed by equations:

- Positive real roots mean exponential growth;
- Negative real roots mean exponential decay;
- Complex roots mean *oscillations*.

Still to come...

We are currently doing differentiation. Next we will be looking at some related concepts: power series and limits followed by partial differentiation, i.e. differentiation of function of more than one variable. We'll then move on to complex numbers, finishing the semester with vectors.

We are currently doing differentiation. Next we will be looking at some related concepts: power series and limits followed by partial differentiation, i.e. differentiation of function of more than one variable. We'll then move on to complex numbers, finishing the semester with vectors.

The next full-class lecture takes place in Week 8 of Semester 1.

See you then.

We are currently doing differentiation. Next we will be looking at some related concepts: power series and limits followed by partial differentiation, i.e. differentiation of function of more than one variable. We'll then move on to complex numbers, finishing the semester with vectors.

The next full-class lecture takes place in Week 8 of Semester 1.

See you then.

Reminders:

- No classes in Week 7
- email address mas-engineering@sheffield.ac.uk
- website <http://engmaths.group.shef.ac.uk/mas140>
(also accessible through MOLE).